HOW TO UNDERSTAND WITTGENSTEIN'S REJECTION OF THE RELATIONAL IDENTITY?

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Abstract. In his text, Tractatus Logico-Philosophicus Wittgenstein articulated an explicitly negative opinion on the sense of relational identities, which are an inevitable tool in almost all mathematical theories today. More precisely, his viewpoint is that relational identities are logical nonsense. Should this viewpoint be accepted as justified, a lot of important content of contemporary mathematics would have to be rejected as meaningless. This paper aims at showing that Wittgenstein's attitude can be understood as conditional, only if some specific positions proposed by Leibniz and Russell are accepted.

Keywords: Wittgenstein, relational identity, Leibniz, Russell.

1. INDIVIDUAL AND RELATIONAL IDENTITY: INTRODUCTION

Generally speaking, the concept of identity is in the ordinary language understood in at least two ways. In one sense, this term refers to a *set of characteristics* that define an entity. Thus, for example, when we speak of national identity, we say that it is defined by that nation's language, culture, customs, territory, etc. Similarly, when we speak of someone's identity, we refer to the features of that person's physical outlook, mental characteristics, fingerprint or ID card. In other words, the identity of a nation or a person can be described or expressed by characteristics such as these. When referring to this kind of understanding of the concept of identity, we will hereinafter use the term *individual* identity. On the other hand, identity can be understood also in a different sense, as a relation that exists between two entities.[1] Hence, for example, we say that two twins between whom we cannot see any visual difference are identical, that is, we say that their physical appearances are identical. Or, when we try to explain someone's

[1] In this context, the terms *entity*, *object* or *thing* will be used interchangeably, as synonyms.

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inconsistency in practical actions, we state that the person in question does not act identically in all the identical situations. Hereinafter this kind of understanding of the concept of identity will be referred to as *relational* identity.

This sort of dualistic conception of the idea of identity can also be found in mathematics, although in somewhat more exact circumstances. Namely, mathematics today makes a distinction between at least two meanings of identity. Most frequently, we speak of the relational identity in the context of stating that there exists, or that it needs to be proved that there exists equality. For example, in elementary algebra, we say that for the arbitrarily chosen variables a, x, n and k there is an identity:

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$$

By this, we state that regardless of the choice of the variables in the above equation, the left-hand and the right-hand side will always have the same value. Similarly, when in the mathematical analysis we speak of the identity

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

we, in effect, state that the value of the left-side expression will always equal the right-side sum regardless of the variable x. Therefore, the relational identity in mathematics is a trivial equation[2], a proposition that asserts certain equality and that is correct for all the values of the variables contained in it. The symbol "="[3] that can be found in all identities can in this context be understood as a mathematical relation.[4]

The latter of the two examples offer identities in which different expressions are on the different sides of the equalities. These identities reveal new expressions, new modes of writing down a mathematical object. In the first example, the object is a function with three variables, whereas in the second it is a function with one variable. Thus, the notion of identity in one mathematical sense of the word is a proposition of the equality of two expressions, that is, a proposition that those expressions represent the same thing. In a general logical form, the notion of relational identity can be found in Russell. According to this definition, two propositional functions x and y are identical if and only if they fulfil the same predicative functions, that is

[2] The term *trivial* is employed here because the solution to such an equation with a finite number of unknowns is not unknown. Namely, if in the equation which is identity there are n variables, then the solution to such equation is n-tuple $(x_1, x_2, \ldots, x_n)$, in which all the components of n-tuple belong to a certain domain. Obviously, generally speaking, the equations do not meet this condition. For example, the equation with one unknown $x^2 = 4$ is fulfilled with two real numbers.

[3] In some cases, when we want to emphasise the identity between two expressions, we use the symbol "≡".

[4] Every mathematical relation is a subset of the Cartesian product of two sets. In our case, if A is a set of all mathematical expressions, then the relation "=" is a subset of the Cartesian product A^2.

$$(x = y) \leftrightarrow \forall \mathrm{F} (\mathrm{F}(x) \rightarrow \mathrm{F}(y))^{5} \tag{1}$$

The concept of identity in mathematics can be understood in another sense as well, without referring to a relation. To be precise, we can also speak of individual identity. By this, we imply a set of *all* individual characteristics by which an entity is recognized or known. This concept is used in situations when an identity of a thing is described as *such* and *such*, that is, in the cases when we define a new object as an object with certain features. This type of methodology is natural in mathematics when defining a new notion. We define it using the familiar, known concepts or through the concepts considered intuitively clear. For example, in algebra, a binary operation on a set S is a mapping of the Cartesian product $S \times S$ into S, whereas in geometry a circle is defined as a set of the points on a plane which are all at an equal distance from the fixed point – the point we call the centre of the circle. Thus, when speaking of individual identity, we speak of a set of individual characteristics that define a certain object, whereas in the case of relational identity we set conditions, such as possession of certain features, the fulfilment of which decides whether two objects can be called identical or not.

Since identity can be understood as a binary mathematical relation, it is, therefore, as we just saw, also a possible context for defining it – the one in which two objects are used. For instance, if we want to define relation "<" ("less than") on the set of integers, the definition can be formulated in the following manner: "for two integers x and y, $x < y$ is valid if and only if…". This is the way we define the arbitrary binary relation "~", and, hence, the binary relation "=" should not be an exception. It is the same formal background as in the approach employed in the definition (1). However, a question arises: can this approach be questioned when it comes to defining the relational identity? Formally speaking, the answer is no, as the concept of identity can be understood as a particular case within the general notion of mathematical relation. Nevertheless, Wittgenstein and Ramsey offer a kind of non-formal questioning:

> Russell's definition of "=" won't do; because according to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it is nevertheless significant) [6]

> ... we ought not to define identity in this way as agreement in respect of all predicative functions, because two things can clearly agree as regards all atomic functions and therefore as regards all predicative functions, and yet they are two things and not, as the proposed definition of identity would involve, one thing.[7]

The core of Wittgenstein's and Ramsey's objection is contained in the viewpoint that there is no sense in talking about the relation of identity between *two* objects. Indeed,

[5] Alfred N. Whitehead, Bertrand Russell, *Principia Mathematica (to *56)*, London, Cambridge University Press, 1997, p. 168.

[6] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, London, Kegan Paul, Trench, Trubner & Co, 1922, 5.5302.

[7] Frank Ramsey, *Foundations,* London, Routledge & Kegan Paul, 1978, p. 201.

we feel that there are no obstacles in speaking reasonably about x and y in terms that $x < y$, or $x \mid y$, but we feel considerable intuitive difficulties in thinking about two objects x and y in the context of $x = y$. Why? One of the possible reasons is that in the latter case we would not know why these would be *two* distinct objects. Namely, a possible distinction between the objects would be already indicated by the distinction signs. If two objects are marked by different signs, it is therefore not natural to expect that the objects in question are identical.

What does the mathematical practice say about this? The well-known mathematical convention implies that in geometry, for example, different objects in a picture, or, in algebra, different objects in a proof, cannot be marked by the same symbols. Thus, for instance, we shall never use the same symbol for two vertices of a single triangle, or two rings (two algebraic structures) when one of them is the other's proper subset. This is not only an artificially imposed convention but a rule to prevent confusion and imprecision. On the other hand, however, there are no formal limitations to using different symbols to mark potentially same objects (the objects which can possibly be proved identical one to another in a process of proving or during an analysis). Such a case is, for example, when an object is a fixed object of a mapping. For instance, when we map point A by axial symmetry in-plane, where point A is on the axis of symmetry, we get point A_1, in which case we write that $A = A_1$. If we then map point A_1 by central symmetry into point A_2, where the centre of the symmetry is precisely point A_1, the result is $A_1=A_2$, that is, $A=A_1=A_2$. Or, similarly, when the uniqueness of a neutral element in a group as an algebraic structure is being proved, the most frequent proof is the one which presupposes that there are two such elements x and y, while the proof ends with the assertion that, in effect, $x = y$.[8]

Which mathematical reasons allow for one object to be marked by an arbitrary number of different symbols and does this freedom entail imprecision and confusion or, on the contrary, make things clear? In the penultimate example, we could say that the same object on a plane is marked by three symbols – A, A_1 and A_2, and thus we write $A=A_1=A_2$. Indeed, we do speak of one object, but also about the three forms in which it appears, or the three roles it assumes. When marked by A, then we talk about the origin of mapping, of the object that belongs to the mapping domain – the axial symmetry by which the object is mapped into A_1. Since any object P is mapped into object P_1, the fact that point A is by the concrete axial symmetry mapped into oneself does not subvert the possibility to mark it with a new symbol. Namely, point A is in no way "privileged" concerning other points from the mapping domain, even if it is mapped into oneself, because it is just one exceptional case, among an infinite number of cases, in which the original in question is joined by the copy in question. Therefore, there is no reason to use a special signifying procedure in this case that would be different from the procedure employed in other cases. Moreover, if in the mentioned

[8] See, for example, Charles Pinter, *A Book Abstract Algebra*, New York, McGraw-Hill Book Company, 1982, p. 36.

case the copy of point *A* is marked by the same symbol – *A* again – it would not be quite clear whether it refers to the original or the copy of the mapping. Hence, using different signs for identical objects is not only safe in terms that it will not create a confusing situation, but it is also helpful for understanding a specific situation and the role of objects in it.

2. WITTGENSTEIN'S UNDERSTANDING OF IDENTITY

As it is obvious from the above mentioned, the concept of identity, generally but also strictly mathematically speaking, can be understood in at least two ways. We have used the following terms: relational and individual identity. Drawing on the above-quoted comments by Wittgenstein and Ramsey, this paper will consider in more detail the standpoint of the former philosopher, who refutes the rationale of the existence of the relational identity. According to Wittgenstein, it is not reasonable to mark a single object with different signs and it is incorrect to understand identity as a relation:

> Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs.[9]

Using wordplay when speaking of the objects' identity, Wittgenstein speaks of a bijection between the set of all defined (mathematical) objects and a proper subset of the set of all signs. Identity (individual identity) ascribed to the sign which signifies an object is ascribed to the object too. Identity is not created concerning the different appearances of the same object. We can, thus, conclude based on the above-cited Wittgenstein's statement that he allows for the existence of the individual, but not of the relational identity. It might perhaps be thought that, when it comes to the relational identity, he allows only for what is in the ancient logic called the law of the identity, that is, the proposition that every object is identical with oneself, $a = a$. However, this is not the case here:

> Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing.[10]

Hence, putting the equality sign between two things, i.e. between two different symbols marking some objects, is nonsensical. So is writing of the identity sign between the same objects/symbols. This observation shows that not only does Wittgenstein see no sense in relational identities, but he also considers the law of identity, which in one form we find in Plato's *Theaetetus*[11], entirely pointless.

[9] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.53.

[10] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.5303.

[11] SOCRATES: Now in regard to sound and colour, you have, in the first place, this thought about both of them, that they both exist ? / THEAETETUS: Certainly. / SOCRATES: And that each is different from the other and the same as itself? / THEAETETUS: Of course. (Harold N. Fowler (eds.), Plato, Vol. II: *Theaetetus, Sophist*, London, William Heinemann, 1922, pp. 159–161)

Wittgenstein's understanding of variables is not in line with the way variables are used in contemporary logic. Namely, he does not agree that different variables can have the same value. For example, according to his viewpoint, the formula

$$s \rightarrow (p \rightarrow q)$$

is an instance of the formula

$$a \rightarrow b. \tag{2}$$

Nevertheless, formula

$$(p \rightarrow q) \rightarrow (p \rightarrow q)$$

is not an instance of the formula (2).

This mode of understanding the meaning of variables can be seen in the following passages as well:

> I write therefore not "f(a, b) . $a = b$", but "f(a, a)" (or "f(b, b)"). And not "f(a, b) . ~$a = b$", but "f(a. b)".[12]

> And analogously: not "($\exists x$, y) . f(x, y) . $x = y$", but "($\exists x$) . f(x. x)"; and not "($\exists x$, y) . f(x, y) . ~$x = y$", but "($\exists x$, y) . f(x, y)".[13]

The previous propositions do not only show Wittgenstein's effort at proving that the sign of identity is not essential in the logical notation, but furthermore, as M. Marion observed, he tried directly to point to the nonsense of the theorems of the theory of identity in *Principia Mathematica* which propose reflexivity and symmetry of the identity relation:[14]

> And we see that apparent propositions like: "$a = a$", "$a = b$. $b = c$. $\supset a = c$", "(x). $x = x$", "($\exists x$) . $x = a$", etc. cannot be written in a correct logical notation at all.[15]

This is not the only part of the *Principia Mathematica* that is a subject of Wittgenstein's elimination of the relational identity. The axiom of infinity had the same fate. Let us be reminded that according to this axiom if α is any inductive cardinal, there is at least one class (of the type in question) which has α terms. An equivalent assumption would be that, if ρ is an inductive class, there are objects which are not members of ρ.[16] There are various interpretations of this axiom. For example, what the axiom of infinity is meant to say would be expressed in language by the fact that there is an infinite number of names with different meanings.[17] Also, it can be read as stating

[12] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.531.

[13] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.532.

[14] In (Mathieu Marion, "Wittgenstein and Ramsey on identity", in Jaakko Hintikka (ed.), *From Dedekind to Godel*, Dordrecth, Kluwer Academic Publishers, 1995, p. 355) it was correctly noted that the above passage refers directly to the theorems *13.15 and *13.17 from *Principia Mathematica.*

[15] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.534.

[16] Alfred N. Whitehead, Bertrand Russell, *Principia Mathematica*, volume II, London, Cambridge University Press, 1927, p. 203, *120.03.

[17] L. Wittgenstein, *Tractatus Logico-Philosophicus*, 5.535.

that there is an infinity of distinguishable entities.[18] At what points exactly is this hypothesis affected by the idea of elimination of, as Wittgenstein calls them, pseudo-propositions – that is, the propositions with the equality sign? In Wittgenstein's opinion, there is one instance of these propositions that forms a foundation for an easier acceptance of the hypothesis:

> All the problems that go with the Axiom of Infinity have already to be solved in the proposition "$(\exists x).\ x = x$".[19]

Why should this proposition form a justification for the axiom of infinity? Let us try and offer an answer which Wittgenstein may have had in mind. As we have already seen, he thought that the $x = x$ equality, that is, the reflexivity of the "=" relation, does not say anything (new) whatsoever about the object x, it is merely a fact which does not need to be stated and which is simply implied as far as any object is concerned. Using Wittgenstein's manner of speaking, we can say that nothing new has been said, the trivial is clear, if we say that $x \neq x$ cannot be or, in other words, that it is not possible to find an object that would be different from oneself. Since the proposition

$$(\exists x).\ x = x\text{"} \tag{3}$$

can be understood as a statement of the existence of the objects that meet a specific criterion (in this case, it is the condition $x = x$, and it is trivially implied that this condition is fulfilled by an arbitrary object), then it follows that this pseudo-proposition ensures truthfulness of the axiom of infinity. Indeed, the number of objects, regardless of their ontological status, that can be differentiated within an exact scientific discipline such as mathematics, or that can be imagined in a real-life, is infinite.[20] Each one of them will meet the relational identity in the proposition (3) and, hence, exist. Therefore, by eliminating the propositions which contain the sign of equality, the pseudo-propositions, as Wittgenstein calls them, the last mentioned in particular, there would be one reason less for accepting the axiom.

3. (NON)DIFFERENTIATING THE OBJECTS THAT ARE BEING IDENTIFIED ONE WITH ANOTHER: PARTIAL AND UNIVERSAL IDENTIFICATION

In this section, we want to point to some of the possible reasons for Wittgenstein's refutation of the relational identities. We will try to show that his reasons could have been found in Russell's and also in Leibniz's understanding of the concept of identity.

[18] M. Marion, "Wittgenstein and Ramsey on identity", in Jaakko Hintikka (ed.), *From Dedekind to Godel*, p. 355.

[19] Ludwig Wittgenstein, *Notebooks, 1914-1916*, Oxford, Basil Blackwell, 1998, p. 10.

[20] In mathematics, for example, the sets N, Z and Q are the sets that contain an infinite number of objects. Based on a mathematical model, in real life we can speak of, let's say, trains with 1, 2, 3, … wagons and, therefore, imagine an infinite row of physical objects.

According to Russell's definition (1), two objects can be identified one with another if they cannot be differentiated, that is if anything that can be attributed to one object can also be attributed to the other. In this respect, these objects can be regarded as one and not as two objects. Let us now consider, in this context, what is it that we mean by saying that two objects cannot be differentiated? Also, let us ask the following question: what do we mean by anything that can be attributed to an object?

Our perception of objects and, consequently, their differentiation can depend on many things. It can, for instance, depend on purely subjective factors, such as our prior experience regarding similar objects, our motivation and interest in certain details connected to those objects. A little boy who likes fast cars is prone, due to his lack of knowledge and experience, to consider identical all the cars of the similar colour and shape, even though they are indeed very different as they are manufactured in different companies. The boy's attention is primarily focused on the cars' shape, colour and speed. Perception of objects can also depend on practical aims, as well as on certain technical or theoretical limitations. A mathematician will easily identify two angles α and β on the base of an isosceles triangle, thus writing $\alpha = \beta$ even though these are not identical objects. The mathematician will be interested only in the measures of those angles and for him, this will be a sufficient reason for their identification. We can, therefore, say that (non)identification of the objects relies on considering the set of features of those objects which is taken into account, that is, the set which is available for consideration to the person who identifies. It may as well be possible that some characteristics are deliberately taken into consideration whereas some others are neglected, but it is also possible that, generally speaking, some features are not available to the identifying subject. To illustrate the first case, we can say that a bank clerk identifies a person A to be the same as person B because of the same income they have. This is a feature relevant in such circumstances and from which ensue many other features that the clerk in question can take into account when it comes to those two persons. For instance, he can consider the ability of these two persons to cover the university fees or whether they are solvent enough for bank credit, etc. In this case, for the bank official A = B. Likewise, when a mathematician identifies the integers which produce the same remainder when divided by k, he will identify the integer n with infinitely many integers, elements in the set

$$\{\ldots, n-2k, n-k, n+k, n+2k, \ldots\},$$

so that he can write that

$$n = n + k$$

even though k is never 0.[21] The identification can be processed not only on the grounds of one selected feature but also on the grounds of an arbitrary number of selected

[21] To be more precise, with respect to the notation of number theory, a mathematician will write $n \equiv n + sk \pmod{k}$, $s \in Z$. In effect, this is an identification of the objects marked n i $n + sk$. For further

features. For example, a supplier in a construction company will probably evaluate-identify in the same way the articles of the same prize, but also of the same quality level.

As an illustration of the second example – the case in which the identifying subject, for some reason, does not have access to all the characteristics of the objects which are identified – we can think of twin brothers who are so similar that no one from their surroundings can make a difference between them on the grounds of some particular feature they possess. There are, however, at least two characteristics that make them different one from another. Whereas one of them is flap-eared, the other has regularly shaped ears. While one snorts at night, the other sleeps soundlessly. The first difference is not known to anyone, as the brothers both have long hair. The other distinction is also unknown to others because they live alone. We can find many similar examples in the exact sciences such as mathematics. Based on the available texts, today we know that for at least 20 000 years the mathematicians did not have a reason to differentiate prime numbers such as, for instance, 29 and 31 by some of their particular properties. Nevertheless, since the late 16th century, when a Mersenne prime was defined in number theory, the numbers 29 and 31 are differentiated. Namely, 29 is not a Mersenne prime, whilst 31 is.[22]

Thus, the identities are formed based on certain properties shared by the identified objects. Generally speaking, a set of properties about which the identification is processed can be U – universal set, a set of all the properties that can be attributed to the compared objects, or its proper subset S. If it is the latter case, then we can speak of a kind of partial identification of objects, which is identification by a set of features available to the examining subject who performs identification, whether an individual, a social group, scientific community, etc. The above-given examples belong to this type of identification. It is a limited type, conditioned and exactly justified by the reasons on which the identification is based. The condition of identification is precisely defined and every identification of this type can be exactly checked in terms of whether it was justified or not. With no difficulties whatsoever, either mathematical or philosophical, can we check if 7 and 19 are identical objects when it comes to the remainder in a division by 3, or if two persons have the same solvency regarding the requirements for getting credit from a specific bank?

On the other hand, if we claim that the set of properties on the grounds of which identification of two objects is made is a set U, the situation is significantly different. This, let us call it, universal identification is apparently what Leibniz and Russell had in mind when dealing with the idea of identification. It seems that Wittgenstein decided to use immediate consequences of this conception to create his extreme viewpoint –

information on this, see Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology*, New York, Oxford University Press, 1997, pp. 121-122.

[22] According to some sources (see Everett Caleb, Numbers and the Making of Us: Counting and the Course of Human Cultures, London, Harvard University Press, 2017, p. 35), the prime numbers were analysed as a specific set even 19 000 years BC. The so-called Mersenne prime was defined by a French mathematician of the same name at the end of the 16th century. Marsenne prime is a prime number reducible to the form 2n – 1, where n is a prime number. For 31 n = 5, whereas there is no such n for 29.

elimination of the relational identity. Namely, as far as Russell is concerned, the universal quantifier in definition (1) suggests that it describes precisely this kind of identification. According to this definition, if an object x has at least one property that an object y does not have, then we could not write that x = y. Definition (1) and the notion of universal identification bring about the question that turned out to be an obstacle to Wittgenstein's reasonable acceptance of the identity x = y, the question which, however, does not rise when it comes to the partial identification. To be precise, it is not clear which elements of the set U for two arbitrary objects are to be identified, nor if this set can ever be distinctly described. If there are two objects, for example, two propositional functions, two integers or two persons, when considering their features/predicative functions we can never be entirely certain that a set of all their characteristics have been precisely defined. Indeed, just as some detail that distinguishes two physical entities can be subsequently spotted, so can a feature by which two mathematical objects are distinguished be "subsequently" perceived. As for the physical entities, the circumstances are usually generally known, and a more attentive observation enables us to make a difference between, say, two twins or two eggs. When it comes to mathematical theories, however, we deal with natural circumstances related to the historical development of mathematics. Throughout that development, the theories are formed and mathematical objects are defined within them. More specifically, new features of those objects are defined, which renders them even more different one from another – the Mersenne prime being an obvious example of this process. To make this observation more generalised, we can take a look into two arbitrary objects x and y and a mathematical theory T, as they share some common characteristics. It is not easy to establish the existence of or the lack of some additional feature of these objects that can be taken into account relating to any point in the historical development of the theory T. An arbitrary mathematical theory, whose objects are analysed or identified, cannot be presented as an unchangeable and fixed set of notions/objects and statements about them. On the contrary, it is a deductively structured system whose final scope, if it exists at all, cannot be predicted. Defining new objects, new relations between them, as well as new predicate functions of the theory's objects is a process that cannot be foreseen.

Taking into consideration all the preceding arguments, we can say that no subject can completely describe the set U for concrete mathematical objects to be identified. There are no indicators that Russell's definition (1) refers to the notions such as "specific historical moment", "subsequently perceived characteristics", subject, etc. It is more likely that Russell's background idea was Platonist and referred to all the properties of mathematical objects, the objects being understood here as ideal entities the existence of which, as well as the characterisation of which, is not conditioned by causal, temporal, spatial or subjective circumstances.[23] Nevertheless, regardless of the possible idealisation

[23] For further information on Russell's deliberation on Platonist ideas see Bertrand Russell, *The Problems of Philosophy*, Oxford, Oxford University Press, 2001, pp. 52-57. The original idea of Platonism in mathematics is to be found in dialogue *Menon*, where the features of mathematical objects that inhabit eternal, timeless and spaceless World of ideas are seen as familiar to our soul. It comes into our body

implied by Platonism, definition (1) proposes a sort of practical procedure for verifying the identification of objects. In other words, identification still has to be carried out by a subject, as someone sometime has to verify if the concrete mathematical objects possess a concrete property. Since it is never possible to determine the set U for two mathematical objects that are being identified, as it has already been illustrated by the above examples, then it is also never possible to have an effective procedure, either formal or empirical[24], by which to verify the statement on the right-hand side of the definition (1). If it is not possible to determine the set of all the characteristics that can be attributed to x and y, then it follows, according to the Definition, that it is also impossible to establish whether these objects are identical or not.

Just as Russell's definition (1) was a suitable foundation for Wittgenstein's understanding of the notion of identification in a universal sense, as we have already pointed out, so did Leibniz's thoughts on identification further reinforce his standpoint – negation of possibility for universal identification. Namely, according to the so-called Leibniz's general logical principle of The Identity of Indiscernibles, roughly speaking, "there are not in nature two indiscernible real absolute beings".[25] In his fifth letter to Clarke, Leibniz says:

> When I deny that there are two drops of water perfectly alike, or any two other bodies indiscernible from each other, I don't say 'tis absolutely impossible to suppose them but that 'tis a thing contrary to the divine wisdom, and which consequently does not exist.
>
> I own that if two things perfectly indiscernible from each other did exist they would be two, but that supposition is false and contrary to the grand principle of reason. The vulgar philosophers were mistaken when they believed that there are two things different solo numero, or only because they are two, and from this error have arisen their perplexities…[26]

At some places, such as the above quotation, Leibniz seems to treat the Principle as a derived standpoint whose justifiability should be verified. He lists reasons to explain the Principle. The existence of individual/separate objects that are not discernible from each other would be opposed to the wisdom of God, but also contrary to the principle of sufficient reason.[27] On the other hand, Leibniz's texts contain also some points that offer material to conclude that the Principle is an axiom, not a derived statement.

precisely from that world, bringing a memory, among other things, of the basic mathematical truths that describe characteristics of mathematical objects (see: *Menon* 82, 85b).

[24] The attitudes about the empirical basis of logic can be found in both Wittgenstein's and Russell's works (see L. Wittgenstein, *Notebooks, 1914-1916*, p. 128 and A. Whitehead, B. Russell, *Principia Mathematica*, volume II, p. 183).

[25] Leroy E. Loemker (ed.), *G. W. Leibniz: Philosophical Papers and Letters*, 2nd ed., Dordrecht, Kluwer Academic Publishers, 1989, p. 699.

[26] *Ibidem*, p. 700.

[27] Fred Chernoff, "Leibniz's Principle of the Identity of Indiscernibles", *The Philosophical Quarterly*, vol. 31, nr. 123, 1981, p. 131.

> Things which are different must differ in something or must have within themselves some diversity that can be noted. It is strange that men have not applied this most obvious axiom, along with so many others.[28]

In whatever way we may understand the formal position the Principle takes in Leibniz's theory, its essence is the foundation on which Wittgenstein's view on the impossibility of identification of two objects is acceptable. This view, drafted in the previous section, is very similar to the above-quoted Leibniz's words. A natural consequence of this viewpoint, as applied in mathematics, is a refutation of the relational identities as meaningless.[29]

Wittgenstein drew attention to the fact that equations which in mathematics express relational identities can contain undiscovered properties of mathematical entities, just as the physics equations can contain undiscovered properties of physical entities. Both mathematics and physics are certainly the scientific disciplines that evolve during the time. Neither Platonism nor Empiricism assumes that the characteristics of mathematical and physical entities are discovered in advance.

When trivialising the role of individual identity one should be careful. Let us propose, for example, any a and b constants (including mathematical or physical entities) for the following simple proposition in the first order-logic:

$$\forall(x)\, \exists(y)\, (x = y).$$

Introducing the constants, the open form of this proposition is obtained:

$$[(a = a) \vee (a = b)] \wedge [(b = a) \vee (b = b)].$$

Four atomic propositions would demand 24 rows of the truth table. However, we eliminate from the table every row that negates the statement on the logical attribute of truthfulness of individual identity. Truth table is, thus, as follows:

	$a = a$,	$a = b$,	$b = a$,	$b = b$	$[(a = a) \vee (a = b)] \wedge [(b = a) \vee (b = b)]$
1.	T	T	T	T	T
2.	T	$\perp$	$\perp$	T	T

What results is the statement that relying on individual identity is trivial, since the attribute of truth or falsity is classically binary implied in the definition of any proposition. Expression of equivalence using implication and conjunction requires the following proposition and T value entailed in it:

$$\begin{array}{ccccc} (a \leftrightarrow a) & \leftrightarrow & (a \rightarrow a) & \wedge & (a \rightarrow a) \\ \mathrm{T} & \mathrm{T} & \mathrm{T} & \mathrm{T} & \mathrm{T} \end{array}$$

[28] L. Loemker (ed.), *G. W. Leibniz: Philosophical Papers and Letters*, 2nd ed., p. 529.

[29] Still, let us mention that when it comes to identities we cannot speak of a complete consonance of attitudes between Wittgenstein and Leibniz. For example, we have mentioned Wittgenstein's view on, according to Leibniz, first truths, such as the law of identity or contradiction (see L. Loemker (ed.), *G. W. Leibniz: Philosophical Papers and Letters*, 2nd ed., p. 267).

Ascribing $\perp$ value to any of the atomic propositions contained in this proposition would bring into question the characteristic truth tables of logical connectives. The logical connectives are defined by their characteristic truth tables. Questioning the definition of logical connectives would also bring into question De Morgan's laws, for instance. Relying on the individual identity is certain in this case. We can justify the caution when trivialising its role. By introducing non-classical values into the logical system (Łukasiewicz) we are still facing the question: Are those non-classical values equal in identity (equivalent) to themselves?

4. CONCLUSION

This article is an attempt at offering a possible answer to the question contained in its title. We wanted to point to a probable background of Wittgenstein's view on a negation of sense of the relational identities. If the objects in question are identified with each other on the grounds of a finite number of the aimed characteristics, it is then a trivial matter. Such objects are not identical, we identify them based on the finite number of properties, that is, we do not have conditions for complete identification. On the other hand, by assuming the universal identification, which Wittgenstein may have had in mind considering Russell's legacy, two objects still cannot be identified one with another. The impossibility to define precisely the set *U* prevents absolute identification. Leibniz's principle of the Identity of Indiscernibles additionally reinforces this position.

Perhaps the above analysis might appear hypothetically constructed in a certain sense since Wittgenstein did not try much to explicitly refer to the philosophical heritage that he drew from in his texts while eliminating relational identities. Nonetheless, it is hard to believe that he could have passed Leibniz and Russell, not because these authorities on the issue of identity could not have been neglected, but also because their ideas, as we have seen, provide solid support to the project of eliminating relational identities. Drawing on Russell, Wittgenstein could consider legitimate only the universal identification. Using Leibniz's ideas, he was able to justify his statement that such identification cannot exist.