KANT'S THEORIES OF SPACE AND TIME IN 19TH CENTURY PHYSICS AND PHYSIOLOGY

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Abstract. This paper considers the 19th c. dispute between thinkers such as Hermann von Helmholtz, who claimed that modern mathematics and sense-physiology refuted Kant on geometry, and neo-Kantian defenders of Kant, who held that the normative role of geometry in physical science placed it beyond empirical investigation. It is argued that 20th c. critics of Kant's theories of mathematics, by following the latter approach, thereby misunderstand the wide scope of Kant's original claim. They interpreted these earlier debates epistemologically, as concerning the difference between the "natural" and the "normative", whereas the fundamental issue was metaphysical, even if it had important methodological consequences. By contrast, the present paper focuses on the role of the ideality of space and time, while remaining fully within the scientific context.

Keywords: Kant; geometry; space; time; metaphysical commitments; Helmholtz.

INTRODUCTION

For 19th c. natural scientists, Kant's principal contribution to knowledge was his theory of the ideality of space and time. This theory was developed, above all, by his successors in Berlin, and, at least later in the century, in Göttingen. Nevertheless, it had already been fundamentally modified by the 1840's at the hands of Johannes Müller and Friedrich Trendelenburg, and then by their respective students, most importantly Hermann von Helmholtz. These philosophers and natural scientists were open to the possibility that the external world might have any dimensionality, so that the ideal space and time of the subject's experience would merely be a "cut" through this multi-dimensional space. In consequence, the distinction between "physiological" and "transcendental" interpretations of Kant's theory that became standard at the end of the century had little meaning for them. The calibration of internal and ideal space was the first step that any scientist had to make in order to isolate, as Helmholtz put it, "that content of our experience that we intuit as not produced through the self-activity of our

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faculty of representation." This project presupposed a specification of three foundational sciences that *were* so produced: the theory of time [*Zeitlehre*], geometry and pure mechanics[1].

Yet, beginning already in the mid-19th century, this form of naturalized Kantianism came under heavy fire by the philosophers who are now generally referred to as "Neo-Kantians". The dispute centered on Helmholtz's claims that modern mathematics and sense-physiology showed that Kant was wrong about geometry and the theory of space, since non-Euclidean geometries were conceivable, and, therefore, it could not be said a priori which of the various geometries of constant curvature was correct. The neo-Kantians responded that Helmholtz and others simply did not understand that, since the role of geometry in physical science is normative, it is beyond empirical investigation. This same position was maintained by 20th American Kant scholars such as Gary Hatfield[2], and, more recently, Frederick Beiser[3]. The aim of this paper is to refute that criticism, both in its historical forms and in this more recent work, by considering in greater detail the doctrines and arguments that were actually at issue in mid-nineteenth century German academia. It will be suggested that, instead of approaching these authors through the normative/natural dichotomy typical of epistemology, we examine their metaphysical commitments, beginning with Kant's own claims regarding space and time.

The paper has four main sections. In the first, I discuss what I take to be Kant's original and foundational doctrine, namely the claim that space and time are not properties of the mind-independent world. I distinguish this view from the metrical conventionalism developed late in the century, which replaced this far-reaching metaphysical thesis with a metrological one. I suggest that 20th c. critics of Kant's first wave of interpreters thereby misunderstand the wide scope of Kant's original claim. In particular, they view 19th c. debates epistemologically, as concerning the difference between the "natural" and the "normative", whereas the fundamental issue is metaphysical, not methodological. In section 2, I propose a classification of pre-and post-Kantian views on the nature of space and time according to the dimensionality of the mind-independent world.

Newtonians proposed that it was intrinsically 3+1 dimensional. Kant, I suggest, thought that it had no dimensionality, because the things in themselves are intrinsically unrelated simples. Kant's first wave of rigorous interpreters, for instance Trendelenburg and Helmholtz, working within the tradition of generalized mechanics developed by Euler and Lagrange in Berlin, assumed that its dimensionality could be arbitrarily great. For them, four-dimensional space-time was merely a cut through a world of yet-unknown structure. In the third section, I emphasize how this same tradition of analytical mechanics, from 1788 onwards, assumed that all basic mathematics was to be

[1] Hermann von Helmholtz, "On General Physical Concepts", in S. Luft (ed.), *The Neo- Kantian Reader*, London, Routledge, 2015, p. 6.

[2] Gary Hatfield, *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*, Cambridge, Mass.: MIT Press, 1990.

[3] Frederick Beiser, *The Genesis of Neo-Kantianism, 1796–1880*, Oxford, Oxford University Press, 2014.

conducted algebraically, thereby consigning the constructive methods of Kant and his idol, Euler, to history. The circularity objection that underlies most modern versions of the conventionalist-normative defence of Kant's views, beginning with the first critics of Helmholtz, is therefore misplaced. For both Riemann and Helmholtz developed their arguments within analytic, not Euclidean geometry. And Kant, as an Eulerian, also sought to provide foundations for analytic geometry, for which Greek geometry provided only basic *lemmata*. I then turn, in section 4, to the wave of late nineteenth-century neo-Kantian criticisms of Helmholtz's and Riemann's work, who were responsible for the development of the metric-conventional reinterpretation of Kant's theory towards the end of the 19th c.

1. KANT'S DOCTRINES OF SPACE AND TIME IN HISTORICAL PERSPECTIVE

The cornerstone of Kant's philosophy is his thesis that time and space are ideal. Without that assumption, the spatio-temporal order of events would reflect relations among things in themselves. This would damage the critical system on two levels. Since global causal determinism would be metaphysically possible, the will might turn out to be unfree. Conversely, skeptical *doubts* about the existence of this same determinism would be unanswerable, meaning that Kant's proof of his law of causality, intended to refute Hume, would fail. A person who sought to defend our use of such a principle would then have only two options: they could interpret it as a hypothesis about mind-independent reality; or they could argue that we are hardwired to think this way. Since neither of these alternatives can exclude that the world will not conform to our expectations, Kant's theory would then collapse on either speculative metaphysics or physiological nativism. Thus, as Kant himself emphasizes, rejection of the *Aesthetic* entails the failure of the *Analytic*.

For this reason, the first wave of his interpreters, in East Prussia, Berlin, and Göttingen, took it for granted that the Aesthetic propounds the fundamental claim of the *Critique*. Kant, his contemporaries and immediate successors all saw him as someone concerned with providing logical foundations for the programme of analytical mechanics that characterized physics in Berlin and Göttingen in the 19th c., the very context in which authors such as Helmholtz, Carl Friedrich Gauss, Felix Klein and Bernhard Riemann worked. They interpreted his work as that of an Eulerian, who was trying to reconcile Newtonian absolutism to Leibnizian relativism by means of a radically new theory of space, time and motion[4]. As such, they took it to be obvious that the theories of space and

[4] For instance, during his lifetime, the German translator of Euler's *Institutiones calculi differentialis*, Johann Michelsen, prefaced the translation (*Leonhard Euler's Vollständige Einleitung zur Differential-Rechnung. Erster Theil*, in Johann Michelsen (ed. and tr.), Berlin, Lagarde und Friedrich,1790) with a long, if tedious, philosophical introduction, linking Euler's work point-to-point with Kant's theory of mathematics, and thereupon sent a copy to the aging Kant. Michelsen taught mathematics at the Gymnasium zum grauen Kloster

time were fundamental to the project, and that the ideality of time, in particular, was a reflection of Kant's Leibnizian background. Whatever the interest of the *Dialectic*, and the doctrine of transcendental freedom, it all comes to naught if time is real. For, Leibnizian monads are timeless and closed individuals, and while they may well have a complex inner structure, it cannot, in principle, be known by another (timeless and closed) monad. The moment that there are genuine causal interactions between substances in time, this form of Leibnizian isolationism is no longer tenable.

By contrast, nineteenth-century interpreters were skeptical of the doctrines of the *Analytic* precisely because they seemed to depend on quite arbitrary features of Kant's "General Logic", which the author admitted having cribbed from 18th c. logic manuals. Writers such as Lange and Frege took Kant's logic to be badly defective, yet they continued to see value in a transcendental approach. That is, they did not abandon the view that, because our access to things in themselves is blocked, reason can construct systems for thinking about the world that will be binding on all future experience. In contrast to their 20th c. successors, they took the *Analytic* to stand in need of almost complete renovation, but their justification for holding that view was that the claims of the *Aesthetic* were, in the large, correct, even if Kant had been wrong about the status of Euclidean geometry.

Today, by contrast, many philosophers take an exactly contrary view, arguing that Kant's claims concerning geometry are defensible, while the thesis of the ideality of time and space is absurd. It is customary to oppose Kant's "transcendental" theory of space and time to its degenerate successors, that is to say the "physiological" interpretations of that theory proposed towards the beginning of the 19th c. by thinkers such as Trendelenburg, Lotze, Müller, and Helmholtz, as well as, later in the century, by Lange. The *Aesthetic*, to the extent that it is read at all, is of interest only insofar as it sheds light on the notion of synthetic *a priori* truth, and only the sections on space are discussed. Leading commentators, such as Paul Guyer, deny that it contains a parallel doctrine concerning time, while asserting simultaneously that if it did, it would be worthless[5].

It is during this same, recent past that the *metaphysical* content of Kant's theory of space and time—that they are properties of the mind, and not of the world— has been largely replaced by a quite different doctrine, namely the *metrical conventionalism* of Henri Poincaré. In this sense, one can argue that the 20th century took the value, if any, of Kant's book to lie in the *Transcendental Analytic*, while the *Aesthetic* itself is an embarrassing relic to be purged, or, still better, ignored. This is a most peculiar development, when one considers that, over this same century, a large portion of the physical commu-

in Berlin from 1778 until the end of his life in 1797. The Berlin elite who attended this Gymnasium during the last decades of the 18th century was, in other words, taught Eulerian mathematics from the point of view of Kant's *Critique*. And this was, indeed, correct, for Kant owned works of Euler's and he demonstrably used these works at a quite sophisticated level when formulating key passages of the *Metaphysical Foundations of Natural Science*.

[5] Paul Guyer, *Kant and the Claims of Knowledge*, Cambridge, Cambridge University Press, 1987, 345f, p. 375. Guyer argues that there is no science parallel to geometry in the case of time, that Kant's theory of time is almost entirely derivative of that of space, and therefore merits no separate treatment.

nity came to see both classical space and time as human constructs. In other words, opposition to Kant's thesis of the ideality of time and space among philosophers has been most pronounced during the very period in which it was finally confirmed. Writers such as Hatfield and Beiser followed this neo-Kantian approach, which developed as a counter-reaction to the physiological tradition. They took the content of the *Aesthetic* to be "transcendental" and, to use Hermann Cohen's term, "negative". But this "transcendental" theory of space is in fact the metrical conventionalism just alluded to, while the physiological theories that are rejected are those which stand closest to Kant's actual view. Contemporary interpreters who approach Kant in this way are, in other words, using a family of concepts and principles that evolved in the late 19^{th} c., and which should have been seen as decisively refuted by 1915 at the latest, as I shall explain in the following.

Since the split between the two approaches played a decisive role in the later polemics against psychologism that defined philosophy's position within late 19^{th} century universities, the standard arguments are still familiar today. On the one hand, one has the programme of empiricist naturalism, possible augmented by Darwin's theory, which argues that human thought, the object of Aristotle's *Psychology*, is a physical process like any other, and should be studied with the methods of biology, sense-physiology and psychology. Arrayed against this approach are the works and arguments of transcendental foundationalists, such as Cohen and Frege, who objected that psychological laws of thought merely describe how people do think, whereas the objects of logic and geometry are *normative.* Logic, construed as the science of the "laws of thought", should not *describe* how people *do* think, but *prescribe* how they *ought* to. To confuse the two is to commit a naturalistic fallacy, which can arise within any of the normative sciences, including geometry, logic, ethics, and, according to some, economics[6].

From this vantage, the works of early naturalizing Kantians reflect trivial misunderstandings, which can indeed be dismissed on *a priori* grounds. Commenting on Hermann von Helmholtz's detailed analysis of the epistemology of geometry in his 1878 *Rektoratsrede*, "The Facts in Perception", Frederick Beiser summarizes their error as follows:

> On the whole, Helmholtz's 1878 lecture, for all its interest and importance, has to be judged a failure. Its shortcomings derived less from any particular doctrine or argument than from its entire programme, from its attempt to base Kant's philosophy upon science. That had not only led to a distorted and contorted interpretation of Kant's theory of space, but it had also misunderstood the aims of Kant's transcendental philosophy, which cannot be grounded on natural science if its purpose is to investigate its very possibility. As Helmholtz spoke in Berlin on that August day in 1878 his programme had already grown old. The younger generation of neo-Kantians – Otto

[6] As is well known, post-war American philosophy came to invert this relation again, in that thinkers such as Quine, who had ingested the naturalist view indirectly from pragmatists such as Peirce, rejected the neo-Kantian foundationalism of Carnap and Wittgenstein, arguing yet again that there are no necessary thought-connections and that foundational questions concerning the structure of thought must only be approached through empirical psychology.

> Liebmann, Hermann Cohen and Wilhelm Windelband –had already challenged his interpretation of Kant and the entire programme behind it. We shall eventually see how, at their hands, the Helmholtzian programme came to grief.[7]

The problem with this research programme, in other words, was that it failed to understand that the a priori conditions of empirical investigation – logic, geometry, and the theory of time – cannot themselves be empirically investigated. If logic is presupposed by all thought, then any attempt to "verify" it would require that we already accept its normative authority. Similarly, to verify empirically which of two geometries was valid for physics, as Helmholtz and Riemann had suggested one might, is to saw at the branch one is sitting on. Poincaré, who put this argument in its canonical form, concluded that our use of Euclidean geometry was therefore immune to empiricist criticisms. In order to enquire into the validity of a geometry, one would have to take into account the possible actions of distortive forces, yet the concept of such forces is parasitic on one's kinematics, thus the investigation would have to begin by *assuming* the validity of one of the geometries under investigation[8].

Helmholtz would be – and was – deeply puzzled by this type of objection. He took his work on the foundations of geometry to represent an extended investigation of the conditions of the possibility of spatial measurement. This was clearly a transcendental investigation, in Kant's terms – one which eventually led, through his contacts with the young Felix Klein, to the Erlangen programme, group-theory, etc. Had he lived slightly longer, he would no doubt have taken the opportunity to refute Poincaré by pointing to Einstein's 1915 publications. For, barely fifteen years after Poincaré made his claims, Euclidean geometry had been finally rejected on empirical grounds. How is it then, that a century later, the programme of investigating Kant's claim that geometry was synthetic *a priori* has come to be seen as a failure?

Note first that Beiser is not referring to the above-mentioned research programme, whose "particular doctrines and arguments" he regards are irrelevant. Rather, his claim that Helmholtz's programme came to grief rests on the standard neo-Kantian objection that Helmholtz confuses a normative question (*quid juris*) with an empirical one (*quid facti*). Whereas the question that drove Helmholtz was not *whether* science involves metrological norms, but rather *which* norm to apply, and according to what criteria? To reply to someone with such concerns that they fail to understand that metrology is normative is like reassuring a man who has been condemned on false evidence that his sentence was, by definition, a case of *jurisprudence*. It is to argue that since the observance of *some* norm, even one arbitrarily chosen, is a condition of the possibility of judgment, or of measurement, no such norm can ever be in error. In the following section, I will propose a different way of categorizing the authors mentioned above, which rests on different metaphysical and mathematical interpretations of the theories of space and time that emerged over this period.

[7] F. Beiser, *The Genesis of Neo-Kantianism, 1796–1880*, p. 205.

[8] See the concluding discussion of this paper and footnote 20 for similar claims regarding the dependence of time-measurement on conventional geometry.

2. THE DIMENSIONALITY OF THE MIND-INDEPENDENT WORLD

In particular, I will propose the following classificatory model. Whereas a realist interpretation of Newton's physics ascribes a four-dimensional structure to the world, Kant ascribes it zero-dimensionality. His 19th century successors, by contrast, working within the programme of generalized mechanics, believed that it might have many more spatial dimensions than Newton, perhaps indeed an infinite number, just as they believed in the reality of motion. Since, for them, three-dimensional spatial intuition was, quite literally, a projection-plane embedded within a higher-dimensional manifold, it could be approached from two different, though ultimately equivalent, directions.

For the researcher who experienced the world subjectively in this way, knowledge of the topological and metrical characteristics of the spatial and temporal manifolds was a first step towards understanding the mind- independent systems whose effects appeared within them. But this same researcher, having constructed a physical model of the external world, would inevitably come to view these same manifolds from another angle. Just as we first learn about colours as elements of our subjective experience, and later come to understand that they are generated by our physiology, so the physiologist of space will eventually redescribe the space of his subjective experience as "due to the self-activity of our faculty of representation". To tell such a person that his first concern – to understand the metrical properties of his representational space – is *undermined* by his conviction that this space is subjective is then a curious *non sequitur*. It must seem – and it did seem to Helmholtz – as if someone had criticized his work on the metric of the colour-space by saying that because colours are the *condition of* (visual) *experience*, the structure of the colour-space cannot be investigated by *empirical* means.

While this characterization may seem loaded against the neo-Kantian critic, it actually points at the true kernel of the latter's position. For, one's natural response is to argue that there is an absolute difference between the manifolds of space and time, on the one hand, and perceptual manifolds like the colour-space, on the other. The latter, as a sensory manifold, is empirical and material. Whereas the theory of *pure* intuition has a transcendental function. It explains the possibility of the a priori sciences of time, space and motion (Phoronomy), and, since these are foundational for physics and for all sciences that rest on physics, they (and they alone) cannot be investigated *by means of* physics. To make the same point intuitively: when our analysis of the colour-space is complete, that space will not form part of the description, for the latter will talk about the physical causes of colours – wavelengths and amplitudes – and it will talk about the neurological structures that map light stimuli onto neural states (nerve energies), but neither of these will be described *in terms of* colours. Whereas, the manifolds of space and time cannot be made to vanish in this way. To use the metaphor popular throughout the 19th century, we can take off our colour-spectacles, but not our spatial ones.

While the young Helmholtz certainly did not deny a foundational role to the sciences of time, space and motion, he and his colleagues still did not draw this conclusion, for the reasons laid out at the opening of his first two papers (1868a, 1868b) on geometry: by using analytic methods, we can free ourselves from the apparent circularity just described. Employing the best physical theories at our disposal, we describe physical systems in the external world and then seek to characterize the physical meaning of perceived spatial intervals in terms of their functional dependence on differences between states of their physical stimuli. Since both descriptions will be algebraic, neither description depends essentially on the form of spatial perception any more than, in the case of colours, the descriptions were dependent on our subjective experiences of colour. The algebraic descriptions of the external systems are inevitably provisional, and they will change as physical theory evolves. But this is not an obstacle to progress – on the contrary, it is the typical case in empirical science.

Since the research programme Helmholtz imagined is alive and well, having scored some recent successes of just the sort he predicted, it is unnecessary to speculate on whether such a programme is *feasible*. On the contrary, its success allows us to see quite clearly why the circularity charge is intuitively compelling. For, one can argue, further research into the physiology of spatial perception is indeed irrelevant to physics, just as predicted. Questions regarding the metrical structure of space-time are not investigated through sense-physiology, just as, to the extent that someone investigated our perception of that structure, the results would be largely irrelevant to space-time physicists. So, the kernel of truth in this criticism of the physiological approach to spatial intuition would be that this manifold plays a foundational role for *all* science, whereas similar physiological structures (e.g. the manifolds of colours, or of acoustic tones) do not. Thus, they are immune to such a circularity objection, while the theory of space is not.

Note, however, that since the subjective manifold of Euclidean three-space is *not* identical with what theorists such as Helmholtz and Gauss took to be the framework of the mind-independent world, the distinction just drawn is not absolute. Assuming that both the world and the subject's perceptual apparatuses are characterized algebraically, the special status of the spatial metric can be maintained only to the extent that it plays a constitutive role in the description of the mind-independent world[9][10]. And, once our foundational principles are purely algebraic, it is only the theory of pure quantity that plays such a role. In other words, once the visual and tactile space has been degraded to being merely psychological, there is no longer any reason to seek foundational principles in its structure, and the free choice involved in our constructing a geometry reflects our new-found (if quite un-Kantian) independence in this regard. Questions can of course arise concerning the suitability of this choice for achieving our goals; however, they will not be settled by means of physiology or introspection, but rather by considerations of

[9] In his first publication (1868a) on the topic, Helmholtz overlooked the possibility of pseudospherical space, and did indeed conclude that Euclidean geometry was constitutive in this sense – no alternative could be constructed. However, he quickly reversed himself on this question, arguing later that the existence of pseudospherical geometries proved that Euclidean geometry does "not follow from the general concept of an extended quantity of three dimensions and of the free mobility of bodies…" (H. von Helmholtz, "The Origin and Meaning of Geometrical Axioms I", in *Mind*, vol. 1(3), 1876, pp. 301–321, p. 314).

[10] Schiemann points out – correctly in my view – that such a constitutive role can be ascribed to the differential, which is in the limit Euclidean. A similar role could be ascribed to the Hilbert space (Gregor Schiemann, *Hermann von Helmholtz's Mechanism: The Loss of Certainty*, Dordrecht, Springer, 2009).

economy (*Zweckmässigkeit*), within logical constraints (*Zulässigkeit*). The foundational component is thereby reduced to logic and algebra, considered as the pure theory of quantity.

3. THE ROLE OF ALGEBRA

Trendelenburg, Müller, and other early 19th century Kantians of a scientific bent pursued a realist version of his philosophy, which did not deny the reality of time, accepted the Law of Causality as a condition on realism, while denying the existence of Newtonian absolute space. Helmholtz, as the most prominent of all of these naturalizing Kantians, was unrelenting in his criticisms of Kant's doctrine of space, not because he was either a naïve realist, nor a naïve empiricist, but because he thought it quite likely that the world had more than three spatial dimensions. This view was a natural consequence of the theory of generalized coordinates that Euler and Lagrange had developed in Berlin, meaning that physicists of Helmholtz's generation no longer worked within a simple Cartesian three-space. We can pinpoint the event that separates Kant from his immediate followers within the natural sciences, namely the 1788 publication of Lagrange's *Analytical Mechanics*. Lagrange, who was Euler's hand-picked successor at the Académie in Berlin, announced his break with all his predecessors, including Euler, in the Avertissement: "One will find no figures in this work. The methods that I develop require neither constructions, nor geometrical nor mechanical reasoning, but only algebraic operations, subject to a regular and uniform pace [*marche*]."[11]

In other words, whereas both Euler and Kant had ascribed evidentiary value to "constructions" of the basic magnitudes of physical theory (space, time and motion), the next generation took it for granted that the language of science, including geometry, was algebraic. From this point forwards, the tools of the scientific foundationalist were almost exclusively conceptual, and the appearance of schemata, figures, and constructions in Kant's theories of mathematics and physics were seen to be outmoded. Helmholtz, like Trendelenburg before him, took it for granted that the foundational science was what had previously been called the "pure theory of quantity", namely algebra. And so, for him, while Kant was entirely correct in worrying that the mathematical structure of the mind-independent world was not Newtonian, the philosopher was entirely *mistaken* in thinking that this form of spatial intuition could yield principles that were anything more than provisional. Indeed, Helmholtz emphasized that the importance of analytic geometry to his geometrical investigations was that it freed us of any dependence on intuition. For him, in other words, there was virtually no difference between the case of colours and the case of visual space. Thus he differed metaphysically not only from Kant, but just as well from the later neo-Kantians who criticized him.

[11] The last sentence puns on Lagrange's and Frederick the Great's shared obsession with clocks, and the regular conduct of one's life. Thematically, it emphasizes that the ultimate standard of regularity is the conceptual notation of algebra itself, here compared to a regiment (Joseph-Louis Lagrange, *Méchanique Analitique* [sic.], Paris, Desaint Lagrange, 1788, vi).

Returning to the taxonomy introduced above, we can easily see where the difference to Kant lies. Helmholtz, as a student of Müller and Trendelenburg, took it for granted that there was a mind-independent world of unknown dimensionality, and that we could come to know about its structure through experimentation, because we could causally interact with it. Kant, by contrast, admitted the existence of this world, but maintained that it had no cognizable structure. Kant's things in themselves are, one might say, unrelated *X*'s, all of whose knowable properties are subject-relative. Typical of such moderate nominalism, Kant does not claim that we know that these X's *lack* intrinsic properties, merely that supposing they do is an empty gesture. For, all their empirical characteristics derive from our awareness of their effects on our sense-organs and our measuring instruments. Similarly, while Kant does not prohibit applying the Law of Causality to objects outside of spatio-temporal intuition, he holds that all "unproblematic" applications of the law require that causes and effects be situated in the spatio-temporal manifolds. Thus, there is no way of reasoning, even inductively, to the *underlying*, non-spatio-temporal world, even if, for all we know, the noumena are monads with complex inner structures. The phenomenal world of spatio-temporal events, on the other hand, is entirely knowable, since it is deterministic and causally closed.

Physiologists such as Helmholtz and his circle, the *Physikalische Gesellschaft zu Berlin*, were, in this sense, aiming to complete a project that had been on the table since Descartes. They wanted to describe the workings of the human organism, including thought itself, in mechanical terms. And, in doing so, they were not targeting Kant or Leibniz, but rather Descartes and Berkeley – thinkers who believed in a human soul, distinct of matter. Kant himself expected that such a theory would come to fruition, for, indeed, it would make little sense to write the *Critique of Pure Reason* unless one did.[12] Because they were realists, they took it for granted that there were correspondence-relations between intuitional representations and the physical systems they imaged. In an early phase of research, we may have to accept the internal structure of these representational systems "as is", for instance when we take it for granted that differences in shades of colours represent objective relations in the world (e.g. relations between different properties of light), without being able to say in advance exactly what these are. But, as our knowledge of light and of our physiology progresses, we may identify exactly what this relation consists in. At the final stage, we may completely reduce our experience of the colour-space to the actions of light on receptors with known sensitivities. Yet we will continue to inhabit a coloured world, and the relations between colours will remain a fixture of our experience, just as they were before the investigation began.

Furthermore, the physical science that preoccupied Helmholtz and the coming generations was electromagnetism. Thus he did not believe that Newtonian mechanics provided a final representation of things in themselves, and had no intention of limiting himself to Kant's closed set of conceptual resources. Classical space-time, even in

[12] Cf. *KpV*, Ak. 5.99.

Kant's relativistic form, was a representation-space forced on us by our physiology, but that did not mean that we could not transcend it. Doing so, however, meant bootstrapping. For, in order to discover the underlying physiological mechanisms that might explain, for instance, how we came to perceive a higher-dimensional world as merely three-dimensional, we would *first* have to construct a mechanical theory within Lagrangian mechanics, *then* do the sense-physiological research within that mechanical framework, and eventually, by shuttling back and forth, adjust our assumptions concerning both domains. For such 19^{th} century thinkers, which included a large number of "phenomenological" physicists up until the move to atomism at the turn of the century, idealism was a conservative heuristic, since they *knew* that they did *not know* what the fundamental constituents of reality might look like.

In sum, for scientists working within the Berlin physical tradition[13], foundational questions were to be settled by algebraic methods, while questions concerning what Kant had called the "pure forms of intuition" were to be investigated empirically. It thereby became quite common to endorse a dual perspective on the world, as exemplified, for instance, in the concluding sections on Helmholtz's last (1878) paper on geometry, where he mounts his argument twice: first in the voice of realism, second in that of phenomenology. The latter perspective stays within subjective experience, and regards the mind-independent world as real and knowable, even if filtered by the mathematical framework we apply to it, including time, space, and the law of causality. The former perspective takes that framework as a provisional description of the mind-independent world, and seeks to understand the thinking subject as a physiological object embedded in that mechanical world. This means, in turn, that the "validity" of intuited relations between internal states can be investigated from two points of view. From the realistic, physiological one, to say that an intuited relation is valid means that it reliably reflects a relation between external states (relations between intuited colour-intervals and wavelength-differences; relations between intuited distances and their 'topogenous moments'). From the idealistic, transcendental one, to say that an intuited relation is valid means that it is a reliable predictor of future events (relations between intuited colours and future measurements of wavelengths; relations between intuited distances and the future evolution of systems).

4. THE NEO-KANTIAN BACKLASH

After his publication, in 1868, of two technical papers concerning the foundations of geometry, Helmholtz became embroiled in disputations with Kantian philosophers across Europe, which led to two subsequent publications focused on their objections. The most sophisticated and influential of these papers was published in 1871/72 by Otto

[13] By this I mean the research programme at the Berlin Academy of Sciences that was initiated by Euler, and carried forward by Lagrange on the mathematical side, and Lambert and Kant within philosophy of science.

Liebmann, one of the philosophers who, according to Beiser, sealed Helmholtz's fate. Helmholtz, in an 1876 edition of his popular scientific lectures, substantially rewrote an earlier paper to counter the objections of Liebmann and others. This same article was published in *Mind* the same year, and occasioned a response from the Dutch philosopher, J.P.N. Land. The general pattern of neo-Kantian objections is already clearly evident in Land's paper, which begins by pointing out the "fundamental error… of positive science", which is that in physics "we must adopt a standard of truth, which in philosophy is the very thing to be settled"[14].

Liebmann's 1871 "On the Phenomenality of Space" was somewhat more sophisticated, and does indeed seem to have affected Helmholtz enough to inspire its own rejoinder, in the form of his (1876) "The Origin and Meaning of Geometrical Axioms I". For, in that paper, Helmholtz specifically attacks philosophers who would claim that, despite his purely algebraic proofs demonstrating that there are several possible systems of homogeneous spatial magnitudes (geometries of constant curvature), one can still defend the position that *human* intuition is necessarily Euclidean. That position was explicitly advanced on the last pages of Liebmann's paper, where he summarized his view in four theses: (1) the space of sensible intuition is a subjective phenomenon, (2) the pure space of Euclidean geometry is an intellectual construction, (3) we do not know if the transcendent (mind-independent) organization of the world is "commensurable" with our spatial intuition, (4) it can be affirmed with certainty that "within our consciousness, which is bound to this spatial intuition, we intuit empirico-phenomenal things and events, with regard to their size, shape, position, direction, distance and speed, in just the way that it occurs in every intelligence that is homogeneous to us"[15]. That is to say, Liebmann, who himself acknowledged that Gauß and Helmholtz held it possible that the mind- independent world had more than three spatial dimensions[16], nevertheless did not conclude from their work and that of Riemann that the non-logical content of geometry[17] was due to experience. Rather, he took these analyses to strengthen Kant's claim, in that they clearly displayed the fact that our form of spatial intuition was on the one hand a *contingent* property of the human organism, in the sense that we could imagine intellects that lacked it, while it remained a *necessary* property of our representations of the world.

As stated, the problem is essentially Cartesian. We have certain inner representations, and we have an outer, transcendent world. Among the inner representations are those of geometry, whose truths are known clearly and distinctly, but from this it does not follow that the material world, should it exist, conforms to these geometric truths. As Descartes did not entertain the idea of alternative geometries, for him this question reduced to the problem of showing that there is a material world characterized by the unique property of extension.

[14] J.P.N. Land, "Kant's Space and Modern Mathematics", in *Mind* 2(5), 1877, pp. 38–46, p. 38.

[15] Otto Liebmann, „Uber die Phänomenalität des Raumes", in *Philosophische Monatshefte* 7 (8):337–359, 1871/1872, pp. 358–59.

[16] *Ibidem*, p. 355.

[17] For instance, Pythagoras' Theorem.

Showing this was showing that "geometry is applicable to experience". In the 19th century version, by contrast, the question is at first glance more straightforward: assuming that there *is* a world that has the structure of a multi-dimensional manifold, is our internal representation of a three-dimensional Euclidean manifold "commensurable" with that world? A subsidiary question thereby emerges, concerning the structure that we find ourselves compelled to ascribe to our *internal* spatial representations: are they necessarily Euclidean, or, as Helmholtz argued, are they partly the result of inductive choices? These two questions correspond to Liebmann's points 3 and 4, above. He concludes that we do not know, indeed we may never know, whether our internal geometry is commensurable with the external one. But we do know that our internal geometry is Euclidean.

It is common, even today, to argue that Helmholtz was exclusively interested in this last question: Can we imagine alternative geometries?[18] But, as was just mentioned, Helmholtz's first two technical papers on the subject are purely logico-mathematical, and explicitly eschew the epistemological question regarding the *source* of our geometrical knowledge. To the extent that anything is "imagined" in these papers, it is imagined by means of algebra. That he came, in later papers, to focus on the question of imaginability is the result of his engaging Kantians like Liebmann and Land, who sought, as we just saw, to draw their line of defence just here. The holder of such a fall-back position (Liebmann) assumes that since the commensurability of inner and outer can never be independently verified (we are stuck in our heads), a proof of the unimaginability of alternative geometries seals the matter.

In fact, Helmholtz was interested in a much more straightforward question: Is Euclidean geometry valid? That is to say, he began to doubt that, as he had argued as a young man, the certainty of "the general or pure natural sciences (theory of time, geometry, pure mechanics) … is an absolute one"[19]. And he had a quite straightforward understanding of what that question means: a geometry is valid when the homogeneous magnitudes that it defines (congruent intervals measured with rigid bodies, congruent spatial figures, etc.) form equivalence classes. Moreover, while it is common to argue, since Poincaré, that the validity of geometry cannot be verified at all, Helmholtz believed that it could be verified twice over, the two different ways corresponding to the two different, yet ultimately equivalent, perspectives mentioned above. Realistically, one could wonder whether the internal geometrical equivalence classes mapped onto external ones. Idealistically, one could verify whether the internal equivalence classes were what he called "physically equivalent" magnitudes. Such magnitudes are those in which the same

[18] Thus De Kock argues that "To get a firm grasp of the stakes of this debate, it should first of all be noted that Helmholtz's primary concern was not theoretical in nature i.e., it did not pertain to the purely logical or mathematical possibility of alternative spaces—but psychological… Given Helmholtz's psychological perspective, his main argument against Kant's a priori account of space (qua Euclidian geometry) pertained to the imaginability [Vorstellbarkeit] of alternative spaces for beings whose powers of reason are quite in conformity with ours'" (Liesbet De Kock, "Helmholtz's Kant Revisited (once More): The All-pervasive Nature of Helmholtz's Struggle with Kant's Anschauung", in *Studies in History and Philosophy of Science*, vol. 56, 2016, pp. 20–32, p. 26).

[19] H. von Helmholtz, "On General Physical Concepts", p. 6

processes unfold in the same time, and in order to ascertain whether this is the case, we do not need to leave the internal domain[20].

Now, when one focuses one's attention on Liebmann's list of properties, one notices a chronological development. The first three (size, shape, position) are the Aristotelian categories that characterize the *res extensa*. The last three (direction, distance and speed), on the other hand, are the determinants of modern kinematics, in particular the content of the Phoronomy of Kant's *Metaphysical Foundations of Natural Science*. These three are, therefore, connected to time. And, while it seems quite straightforward to ask, with respect to the notions of shape and distance, whether external objects are "commensurable" or "congruent" with their internal representations, it is not obvious what it could mean to say that speeds and directions are so. For Kant, of course, the latter question had no more meaning than the first – the mind-independent world has zero-dimensionality, and verifies no metrical relations. For Helmholtz, by contrast, this temporal relation was in fact the *only* sense in which the internal world was "like" the external one:

> We cannot even call sense impressions pictures; for a picture depicts the same through the same. In a statue we represent bodily form through bodily form, in a drawing, the perspectival view of an object through the same in the picture, in a painting, colour through colour.
>
> We can call sensations pictures of the process of events only with respect to the flow of time. Under the determinations of the flow of time falls number. In these relations they therefore yield more than would mere signs.[21]

That is to say, to speak of correspondences, in the sense of similarity, between mind and world is fruitless, unless we are talking about quantitative relations. And quantitative relations amongst our sensations also do not correspond, through similarity, to the external world. Rather, they express measurable functional dependencies between events. For instance, two shades of red do not correspond univocally to anything in the external world—not even wavelengths, for our eyes adjust to lighting conditions. But the

[20] Schlick in (Helmholtz, *Schriften zur Erkenntnistheorie*, in P. Hertz and M. Schlick (eds), Berlin, Springer, 1921, p. 173) and, following him, Hatfield (*op. cit.*, pp. 222, 336), have both argued that this definition of physically equivalent magnitudes is circular because there is no way of defining "equal time" without assuming a spatial metric to begin with. Similarly, Friedman (Michael Friedman, *Kant's Construction of Nature*, Cambridge, Cambridge University Press, 2013) has recently argued that Kant advances no mathematical concept of time in the Transcendental Aesthetic, since the latter can only be defined through the laws of motion and the Principles that undergird them. The view derives from the conventionalism of Poincaré, which the early logical empiricists took to be Einstein's own view. I see no evidence that Einstein held that view—he criticized Poincaré's position as holding only "sub specie aeterni", and often observed that, e.g. "it is no petitio principii to posit the notion of a periodic process … when one is clarifying the empirical content of the time-concept" where "this conception is entirely complementary to the preceding notion of a rigid or quasi-rigid body in the interpretation of space". (A. Einstein. „Physik und Realität", in *Journal of the Franklin Institute*, vol. 221 (3), 1936, pp. 313–347, p. 322). At no point is it suggested that the former depends conceptually on the latter. It seems equally unlikely that Helmholtz, or Kant before him, would have accepted this objection, and not because they were unaware of it.

[21] Helmholtz, 1892, p. 47.

difference, that is to say the temporal variation, between two colours records a difference in the stimuli that caused them. Time, motion and number[22] are the only internal elements that correspond to something external, and it is only in connection with them that an internal geometry can correspond to an external one.

These two perspectives on the question of validity are developed independently in Helmholtz's last paper[23] on the "space problem", which was published in *Mind* as a response to Land, but also as an Appendix to the German publication of the "Facts in Perception":

> II. In this second section I will start from the position that Kant's hypothesis of the transcendental origin of the geometrical axioms may be correct though not proved…. I will also, in the first instance, adhere to the realistic hypothesis and speak its language, assuming that our sensible impressions are caused by things really existing in space and acting upon our senses.…I regard this view of things, however, expressly as hypothetical, and I mean afterwards to drop the realistic hypothesis in my third section, when I will repeat my exposition in abstract language, without any assumption as to the nature of real existence.
>
> First of all, we must distinguish between equality or congruence of space-magnitudes as dependent on the assumption of transcendental intuition, and their equivalence as determined by measurement with physical instruments.
>
> I call physically equivalent those space-magnitudes in which under like conditions and within like periods of time like physical processes take place. The process most commonly employed, with due precautions, for the determination of physically equivalent space-magnitudes is the transference of solid bodies from one to the other, that is to say, measurement with compass and rule.[24]

> III. The discussion in the second section has been confined to the objective sphere, and conducted from the realistic point of view of natural science, whose aim is to comprehend or grasp conceptually the laws of nature. Towards this end perceptive knowledge is either only a mere help or, as the case may be, a false show to be got rid of.…let us now drop out of sight the hypothetical element in the realistic view….The only assumption we still maintain is that of the law of causation, to the effect, namely, that all mental states having the character of perception that come to pass in us do come to pass according to fixed laws, so that when different perceptions supervene we are justified in inferring therefrom a difference of the real conditions determining them.[25]

[22] Helmholtz's views on this question evolved over the years. At the time he wrote his first two papers on geometry (1868a, 1868b) he also argued that only mathematical descriptions could bridge the external and internal; however, he included space among these. By the 1890s, in part because of the arguments he developed in his debates with Land and Liebmann in the 1870s (discussed above), space was eliminated. Yet, even in his early writings, time is privileged over space in this way. Similarly, Trendelenburg (*Logische Untersuchungen*. vol. 1) had insisted that what was common to mind and world was only motion, and that both time and space were merely "factors" of this motion, that is to say, dependent magnitudes.

[23] H. von Helmholtz, "The Origin and Meaning of Geometrical Axioms II", in *Mind*, vol. 3(10), 1878, pp. 212–225.

[24] *Ibidem*, p. 217.

[25] *Ibidem*, p. 222.

Note first the definition of “physically equivalent magnitude”, which is assumed throughout the rest of the paper. Two spatial magnitudes are physically equivalent when identical processes unfold within them in the equal periods of time. Helmholtz had used a similar definition already in his earliest writings on this topic, for instance in his unpublished manuscript, “On General Physical Concepts”, where he defines two objects as *equal* “in a relation if [one] can be substituted for the other everywhere where the result of a combination is considered only with respect to this relation”[26]. Equal times, he continues, are ones “in which the same changes take place under the same circumstances”[27], while equal distances are determined by the transport of rigid bodies. Inertial paths, in turn, are straight lines traversed by bodies in states of uniform motions. That is to say, equal times are the equivalence classes determined by clocks, equal spaces are the equivalence classes determined by rigid bodies, and since the “uniformity” of what he calls “stable motions”, requires that bodies trace out equal spatial intervals of “straight lines” in equal times, equal spatial intervals as measured by rigid bodies are also equal intervals of uniform, or inertial paths.

At the time he wrote “On General Physical Concepts” (<1847), Helmholtz himself assumed that these two definitions – rigid-body congruence, equal segments of inertial paths – would coincide. Even the first versions of his 1868 papers on geometry continue to argue that there is only one possible system of homogeneous spatial magnitudes. By contrast, the fundamental assumption under attack in this mature work from the 1870s is that we know a priori that these two definitions of equal spatial intervals *must* agree:

> Taking the notion of rigidity thus as a mere ideal, a strict Kantian might certainly look upon the geometrical axioms as propositions given a priori by transcendental intuition which no experience could either confirm or refute, because it must first be decided by them whether any natural bodies can be considered as rigid. But then we should have to maintain that the axioms of geometry are not synthetic propositions, as Kant held them: they would merely define what qualities and deportments body must have to be recognized as rigid.
>
> But if to the geometrical axioms we add propositions relating to the mechanical properties of natural bodies, were it only the axiom of inertia or the single proposition that the mechanical and physical properties of bodies and their mutual reactions are, other circumstances remaining the same, independent of place, such a system of propositions has a real import which can be confirmed or refuted by experience, but just for the same reason can also be got by experience. … If such a system were to be taken as a transcendental form of intuition and though there must be assumed a pre-established harmony between form and reality.[28]

That is to say, one can maintain that any arbitrary geometry is “valid”, insofar as one takes it to be a metrological stipulation of what counts as rigid. But, for the

[26] H. von Helmholtz, “On General Physical Concepts”, p. 7.
[27] *Ibidem*, p. 8.
[28] H. von Helmholtz, “The Origin and Meaning of Geometrical Axioms I”, pp. 320–21.

physicist, this is just a first step[29]. As in his manuscript from 1847, the regions of space parceled out by rigid body measurement have a meaning for physics: they must be the segments that a body in inertial motion traces out in equal periods of time; or, put otherwise, they must be the magnitudes that are invoked in kinematics, such as the Phoronomy of Kant's *Metaphysical Foundations*.

Assuming that a stipulated geometry will satisfy this requirement is assuming a "pre-established harmony" between these two different definitions of equal spatial magnitude.

The two perspectives Helmholtz develops at the end of his 1878 paper merely cash out this "pre-established harmony" argument in two different ways. Looked at from the point of view of the physicist and physiologist, the question is whether, and how, we could know "that space-magnitudes equal to one another in transcendental intuition are also physically equivalent"[30], given that, using Beltrami-style projections, we can construct coordinate-systems agreeing to multiple geometries within the same manifold of experience. Looked at from the phenomenological point of view,

> When we observe that the most diverse physical processes may go on during equal periods of time in similar fashion at different but congruent parts of space, the real meaning of such a perception is, that there may be in the sphere of reality equal sequences and aggregates of hylogenous moments combining with certain distinct groups of topogenous moments, which latter we then call physically-equivalent. We may thus discover by observation what special figures appearing in our perception correspond with physically-equivalent topogenous moments; and experience tells us that they are equivalent for all physical processes.[31]

That is to say, under the premise that our perceptions are caused by something (a premise which Kant himself seems to affirm on the first page [B33] of the *Critique*) and that motion is real, the same question – how we know that a stipulated geometry gets physical equivalence right – has an only slightly different significance. From the phenomenological perspective, it now means: How do we know that "the equality of the *perceived distance* and the *physical equivalence* of the distance depend on the same function of the topogenous moments or not?"[32] (my emphasis) Put in the terms of our colour-examples, Helmholtz is therefore asking: How do we know that perceived differences in colour variations always depend on a variation in, e.g., the wavelengths of the causes, which itself counts as equivalent in all other physical circumstances? How do we know that what we perceive as an equivalence class actually will behave like one?

We note, in conclusion, that both of these forms of the argument depend essentially on the concept of a physically equivalent magnitude, which itself depends

[29] This step is strictly analogous to, for instance, the choice of a particular lump of metal and weighing technique in order realize the concept of mass. Whether or not these choices will yield a homogeneous dimension of mass is a second point for reflection.

[30] H. von Helmholtz, "The Origin and Meaning of Geometrical Axioms II", p. 219.

[31] *Ibidem*, p. 224.

[32] *Ibidem*, p. 225.

on the notion of "equal times". Since geometry only becomes meaningful to physics in its connection to time, to ask whether a geometry is valid for experience is essentially to ask whether it parcels the world into magnitudes that are homogeneous in temporal relations. To the extent that someone argues that we can have immediate knowledge of the nature of such equivalence classes, they must explain, when confronted with multiple, and incompatible classes, how this could be known in advance, and this cannot be done, whether one is a realist or an idealist.

5. CONCLUSION

We began with a simple question. How is it that an obviously successful research programme, which changed our understanding both of "internal" and "external" geometries comes to be represented as an abject failure? The criticism, as we saw, is that the researchers in question did not understand that geometry, as a normative science, is immune to any future revision, because, as Land put it, in "physics we must adopt a standard of truth, which in philosophy is the very thing to be settled". But that supposition holds only if one believes that the mind-independent world is fundamentally unstructured, or, more conservatively, that there is *no* structure that is *common* to that world and our subjective experience of it[33]. Kant did make this strong claim, and thus he could consistently argue that his a priori foundations were irrefutable. Since the world is filtered through these a priori structures, there simply cannot be any experiential data at odds with them.

But the moment this assumption is dropped, the possibility of such a defence collapses. If time and motion are, as Trendelenburg argued, common to mind and world, then we can correlate "internal" events and relations with causes in that world, to which end we do not require a final account of its structure. Furthermore, since we can formulate our descriptions of both worlds in terms of numbers and algebraic equations, no vicious circle is implied. Since neo-Kantian critics, both then and now, approach these questions purely epistemologically, they overlook the fact that this very epistemological approach depends on the above metaphysical thesis – the very one addressed at the opening of this paper.

If space and time are not exclusively ideal, then questions of correspondence, congruence and "commensurability" may be posed and answered. This is – in fact – what happened in the 19th century, and the result was a fundamental revision to our theories of the internal manifolds (as reflected in contemporary work on the physiology of space-perception), and, shortly afterwards, a fundamental revision to our theories of the external manifolds as well. And this means, in turn, that the present dislocation of

[33] That Kant did not show that spatial intution must be exclusively ideal is usually referred to as "Trendelenburg's excluded alternative." If I have not discussed this thesis in the preceding, then only because it is only the starting point for Trendelenburg's, Müller's and Helmholtz's programme, albeit one which occasioned rather much discussion within 19th c. Kant scholarship.

these two fields – the physiology vs. the physics of space and time – does *not* support the claim that metrological (or other) norms cannot be empirically investigated and refuted. On the contrary, this dislocation is a direct consequence of the fact that they were so investigated, and this by the very people who, according to the neo-Kantians and their modern apologists least understood the significance of their own work: Helmholtz, Riemann and Klein[34].

[34] It should not be concluded from the above that only scientific approaches to Kant's work are of interest, and still less that only scientists correctly understood his work. The moral is far simpler: those who read the works of scientifically literate philosophers such as Kant must ensure that their understanding of his work is mathematically at least as sophisticated as his own. Unfortunately, this cannot be said of a single major Kant scholar since the 1890s, after which point it became customary to read his mathematics through the Hellenistic tradition.