ON THE OBJECTIVITY OF SCIENTIFIC KNOWLEDGE. MODELS AND THEORETICAL REPRESENTATIONS OF STRUCTURE AND PROGRESS IN SCIENCE. THOMAS KUHN’S LEGACY

WHAT AN INTERPRETATION OF QUANTUM MECHANICS SHOULD BE?

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Abstract. There are many so-called interpretations of quantum mechanics, but we presently do not have any clear-cut criteria to identify them. Usually, scientists do not distinguish the different interpretations of the standard quantum theory from its alternative or rival theories, although there is an established fact that there exist different – even non-empirically equivalent – quantum theories. In this article, I put forward some criteria to distinguish between formulations, interpretations, and alternatives to a given quantum theory. I then show that we have just some partial criteria to identify quantum theories and distinguish them from interpretations. According to such criteria, all interpretations of a given quantum theory must be empirically equivalent to it, otherwise, they are rival theories, and must not be logically equivalent to it, otherwise, they are different formulations of such theory. I conclude that interpreting a quantum theory cannot consist in providing a unique ontology for that theory because the same theory is compatible with many different ontologies.

Keywords: quantum theories; formulations; interpretations; minimal interpretation; empirical equivalence.

1. INTRODUCTION

The standard quantum theory (hereafter SQT) and its standard interpretation took its present formulation in the early 1930s when Von Neumann published his epoch-making treatise[1]. The so-called Copenhagen interpretation (a name coined by Heisenberg in 1958[2]) was not finished by then, but its antirealist stance was already present in most of its early systematic presentations, as those contained in

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[1] John von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Berlin, Springer, 1932.

[2] Werner Heisenberg, *Physik und Philosophie*, München, Piper, 1958.

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Dirac's and Heisenberg's books[3]. Discussions about the interpretation of the newborn theory began as early as 1927, mainly in the famous Fifth Solvay Conference[4]. Bohm's 1952 interpretation was the first alternative to the standard Copenhagen's view[5]. Everett's 1957 interpretation was the second alternative interpretation[6]. Since then, many different interpretations were put forward. No interpretation was conclusively discarded so that nowadays we have a myriad of different interpretations, some of which are undoubtedly rival theories of the SQT. We do not have reached an agreement on the simple question of whether Bohmian mechanics and Everett's many-worlds interpretation are alternative interpretations of the SQT or alternative rival theories. The predicament is that confused for two basic reasons. First, neither scientists nor philosophers of science have reached a consensus on shared criteria to identify theories. Second, we do not know precisely what an "interpretation of quantum mechanics" is and, much less, based on which criteria should we choose between different consistent interpretations of the same quantum theory.

In what follows I will discuss the question of how to identify a quantum theory and how to distinguish between reformulations, interpretations, and rival theories of a given quantum theory. On the way, I will intend to explain what an interpretation is not and what it should be. I do not claim to have definitive answers to those fundamental questions; nonetheless, I believe, some misconceptions can be dispelled.

2. THE PRESENT STATE OF THE INTERPRETATION PROBLEM

If we were to ask working scientists what precisely the problem of the "interpretation of quantum mechanics" is, most will answer they do not have a clear idea about it. Those who work in the domain of quantum physics will probably answer that the problem consists in "understanding" quantum mechanics

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[3] Paul Dirac, *The principles of quantum mechanics*, Oxford, Clarendon Press, 1930; Werner Heisenberg, *Die physikalische Prinzipien der Quantentheorie*, Leipzig, Hinzel Verlag, 1930. To the best of my knowledge, Heisenberg was the first who spoke in his 1930 book about a *Kopenhagener Geist der Quantentheorie* that inspired his account of the new quantum theory.

[4] The very interesting proceedings of this conference have been edited and commented by Guido Bacciagaluppi, Anthony Valentini, *Quantum mechanics at the crossroads: Reconsidering the 1927 Solvay Conference*, Cambridge, Cambridge University Press, 2009.

[5] David Bohm, "A suggested interpretation of quantum theory in terms of 'hidden variables'", in *Physical Review*, vol. 85, nr. 2, 1952, pp. 166–193.

[6] Hugh Everett, "'Relative state' formulation of quantum mechanics", in *Review of Modern Physics*, vol. 29, nr. 3, 1957, pp. 454–462. The complete writings of Everett were edited by J. Barrett, P. Byrne, (eds.), *The Everett interpretation of quantum mechanics. Collected works 1955–1980 with commentary*, Princeton, NJ, Princeton University Press, 2012.

or “making sense” of it. Both answers are vague enough to render the problem ill-posed and, consequently, unsolvable[7].

Besides, the number of the so-called “interpretations of quantum mechanics” has increased in recent decades to the point that no scientist or philosopher of science can feel entitled to list all of them. Very often, the partial lists offered by different authors have only a partial overlap and generally include rival quantum theories under the heading of interpretations. In the loose terms of most scientists – and philosophers alike – they are also called, indistinctly, “theories”, “models”, “interpretations”, “approaches”, “accounts”, “views”, “formulations”, “reconstructions”, “alternatives”, and more. The lists almost always include the Copenhagen interpretation, Bohm’s hidden-variables interpretation, the many-worlds interpretation, the statistical or ensemble interpretation, and the GRW interpretation. Philosophical books often mention and discuss the modal interpretations, generally neglected by scientists, and other more recent interpretations, such as the transactional interpretation, the many-minds interpretation, the relational interpretation, objective collapse interpretations, Quantum Bayesianism, consistent histories, quantum information, $3N$-dimensionalism or wave-function realism, and more. Even a cellular automaton interpretation has been recently added to the growing list[8]. To the best of my knowledge, there is no single book that describes all these proposed interpretations[9].

There is no agreement among scientists or philosophers about which are the interpretations at stake and how should we choose one interpretation over the others. The old Copenhagen interpretation, once prevalent among scientists, today is almost a strategy to evade or circumvent the problem of interpreting quantum physics. The philosophical analysis has made clear that there is no such a thing as a unitary Copenhagen interpretation because the philosophical opinions of its main architects, such as Bohr, Heisenberg, and Pauli, among others, greatly differ

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[7] Many books written by scientists bear those words in their titles, among others, Roland Ommès, *Understanding quantum mechanics*, Princeton, Princeton University Press, 1999; Jean Bricmont, *Making sense of quantum mechanics*; Cham, Springer, 2016; Franck Lalöe, *Do we really understand quantum mechanics?*, second edition, New York, Cambridge University Press, 2019; Detlef Dürr, Dustin Lazarovici, *Understanding quantum mechanics: The world according to quantum foundations*, Cham, Springer, 2020.

[8] Gerard T’Hooft, *The cellular automaton interpretation of quantum mechanics*, Cham, Springer, 2016.

[9] Max Jammer, *The philosophy of quantum mechanics: The interpretations of quantum mechanics in historical perspective*, New York, John Wiley & Sons, 1974 was at the time of its publication a careful and comprehensive review of most interpretations, but it is now out of date, although still useful. Olival Freire (ed.), *The Oxford handbook of the history of quantum interpretations*, New York, Oxford University Press, 2022 is a more recent comprehensive account of many interpretations by different authors. I do not intend to provide here specific references for each interpretation, which are too numerous.

between them[10]. What is presently called the Copenhagen interpretation is rather a family of vague and rather incomplete answers to the some of the philosophical problems posed by quantum mechanics that share a family resemblance. What they have in common is mainly an anti-realist attitude toward quantum physics and a consequent refusal to address ontological questions. However, it is undeniably true that a kind of "spirit of the Copenhagen interpretation", although not very well defined, has permeated the way quantum mechanics is taught. According to this construal, the SQT does not aim at providing a literally true description of the physical world, but it is rather a tool for calculating the probabilities of experimental results. Therefore, the wave function Ψ (the fundamental mathematical symbol of the theory) does not refer to any entity in the physical world; it is solely a device designed to extract probabilistic predictions, through the Born rule.

The SQT, as formulated in the early 1930s by Dirac and Von Neumann, has been until the present day the theory that is taught in the majority of physics courses and the one that is presented in most textbooks, either elementary or advanced. Usually, those textbooks do not address the interpretative problem or devote a passing mention to it in a few pages.[11] Most courses in quantum

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[10] See, for instance, Don Howard, "The Copenhagen interpretation", in Olival Freire (ed.), *The Oxford handbook of the history of quantum interpretations*, pp. 521–542, and the references cited there. Almost every book on the philosophy of quantum mechanics discusses the Copenhagen interpretation from different points of view and often provides different reconstructions of it. See, for instance, Jeffrey Bub, *Interpreting the quantum world*, revised edition, Cambridge, Cambridge University Press, 1997, chapter 7; and Travis Norsen, *Foundations of quantum mechanics: An exploration of the physical meaning of quantum theory*, Cham, Springer, 2017, chapter 6. Mara Beller, *Quantum dialogue: The making of a revolution*, Chicago, University of Chicago Press, 1999 is a detailed historical study of the genesis of that interpretation. Aage Petersen, *Quantum physics and the philosophical tradition*, Cambridge, MA, MIT Press, 1968 is a valuable account of its philosophical foundations.

[11] There are some exceptions, such as Chris J. Isham, *Lectures on quantum theory: Mathematical and structural foundations*, London, Imperial College Press, 1995; Alastair I. M. Rae, *Quantum mechanics*, fourth edition, Bristol, Institute of Physics Publishing, 2002; and Steven Weinberg, *Lectures on quantum mechanics*, Cambridge, Cambridge University Press, 2015, although they address a limited number of conceptual problems and a sparse sample of alternative interpretations. Of course, there are many books written by physicists devoted to the interpretation of quantum mechanics, among them, those by Gennaro Auletta, *Foundations and interpretation of quantum mechanics*, Singapore, World Scientific, 2001 and *The quantum mechanics conundrum*, Cham, Springer, 2019; Jeffrey Bub, *Interpreting the quantum world*, Cambridge, Cambridge University Press, 1997; Jean Bricmont, *Making sense of quantum mechanics*, Cham, Springer, 2016; Bernard d'Espagnat, *The conceptual foundations of quantum mechanics*, Reading, Perseus Books, 1999 and *Veiled reality: An analysis of present-day quantum mechanical concepts*, Boulder, Westview Press, 2003; Michael Dickson, *Quantum chance and non-locality: Probability and non-locality in the interpretations of quantum mechanics*, Cambridge, Cambridge University Press, 1998; Detlef Dürr, Dustin Lazarovici, *Understanding quantum mechanics*, Cham, Springer, 2020; Giancarlo Ghirardi, *Sneaking a look at God's cards: Unraveling the mysteries of quantum mechanics*, Princeton, Princeton University Press, 2005; Gérard Gouesbet, *Hidden worlds of quantum physics*, New York, Dover, 2013; Gregg Jaeger, *Entanglement, information, and the interpretation of quantum mechanics*, Berlin, Springer, 2009; Franck Laloë, *Do we really understand quantum mechanics?*, New York, Cambridge University Press, 2019; Travis Norsen, *Foundations of quantum mechanics*, Cham, Springer, 2017; Roland Omnnès, *The interpretation of quantum mechanics*, Princeton, Princeton University Press, 1994 and *Understanding quantum mechanics*, Princeton, Princeton University Press, 1999; Franco Selleri, *Le grand débat de la théorie quantique*, Paris, Flammarion, 1994. They are not always accurate in the use of philosophical concepts.

mechanics are devoted to training the students in solving technical problems, a task at which the SQT has been extremely successful. Accordingly, there is a robust consensus concerning how to use and apply the SQT. Moreover, scientific practice has shown that using the theory and applying the quantum recipe does not require addressing the interpretative problem nor taking a stance toward a specific interpretation. That is the main reason why working scientists can neglect the interpretative problem in their everyday activities, both in research and teaching. However, the proliferation of conflicting interpretations might even raise doubts concerning the SQT, which, despite its practical success, has been regarded as lacking sound theoretical foundations by some scientists -Einstein, in the first place- and many philosophers of science. After reviewing some interpretations of the theory, Steven Weinberg has expressed his discomfort in these terms:

> My own conclusion is that today there is no interpretation of quantum mechanics that does not have serious flaws. This view is not universally shared. Indeed, many physicists are satisfied with their own interpretation of quantum mechanics. But different physicists are satisfied with different interpretations. In my view, we ought to take seriously the possibility of finding some more satisfactory other theory, to which quantum mechanics is only a good approximation.[12]

The issue of interpreting quantum theories belongs both to the fields of science and the philosophy of science. Nonetheless, many distinguished scientists (such as Richard Feynman and Steven Weinberg) have neglected the problem because they regard it as a purely philosophical one, a problem that stands outside the proper domain of the physical sciences[13]. My view is that it is a very significant problem for physics as an empirical science, and for that reason, it is a question that should not be neglected by working physicists. Granted, the philosophical questions will not make any difference regarding the mathematical calculations or the derivation of experimental predictions from quantum theories. Nonetheless, the philosophy of quantum mechanics can contribute to the understanding of quantum theories and is even capable of exerting an influence on the formulations, the use, and the teaching of quantum physics generally. A division of labor, however, is necessary to avoid the intrusion of purely philosophical issues, which because of their very nature do not admit of empirical answers, into the body of science. In my view, it is not the business of the philosophers of science either to reformulate quantum theories or to put forward new interpretations of them; much less, to build alternative quantum theories. What philosophers of science can do –

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[12] Steven Weinberg, *Lectures on quantum mechanics*, p. 102.

[13] See, for instance, Richard Feynman, Robert Leighton, Mattew Sands, *The Feynman lectures on physics. The new Millenium edition. Volume III: Quantum mechanics*, New York, Basic Books, 2011, section 2–15, and especially Steven Weinberg, *Dreams of a final theory*, New York, Vintage Books, 1994, chapter 7. The book by Detlef Dürr, Sheldon Goldstein, Nino Zanghi, *Quantum physics without quantum philosophy*, Berlin, Springer, 2013 has a self-explanatory title.

with benefits both for physics and philosophy – is to elucidate some very general issues that are often uncritically assumed by scientists, uncover new conceptual problems, and pose unsettled questions.

3. TERMINOLOGICAL PRELIMINARIES

We must begin by clarifying the terminology with which the problem of interpretation should be addressed; in the first place, the ambiguous expression "quantum mechanics" itself. This expression is used sometimes as a *proper name* or *definite description* intended to refer to a single theory, such as is the case with "Newton's mechanics" or "the general theory of relativity". On other occasions, it is used as a *common* or *class name* intended to refer to a specific domain of physics, such as "mechanics" or "electrodynamics". In this sense, it refers to a branch of physics –mechanics- which qualifies as a quantum one in opposition to classical or non-quantum mechanics. In a broader sense, sometimes it is used to refer to quantum physics generally, which includes not only quantum mechanics, but also quantum electrodynamics, quantum chromodynamics, and quantum gravity. Of course, a name cannot be a proper name and a class name at the same time. The use of "quantum mechanics" as a proper name, as the name of a single theory, has created the false appearance that there exists just one quantum mechanical theory (in need or not of interpretation), when in fact there are many alternative theories that belong to the domain of quantum mechanics, and of quantum physics generally. This confusion has not arisen in other domains of physics, where the different rival theories that belong to the same branch of physics are distinguished by using different proper names. For instance, Einstein's general relativity is a theory that belongs to the class of metrical theories of gravitation. Other theories, such as the Brans-Dicke scalar-tensor theory or the Rosen bimetric theory, are rivals of Einstein's theory that belong to the same class of gravitational theories.

There are historical reasons that explain the ambiguities in the use of "quantum mechanics". For many years – roughly from 1927 to 1952 – there was just one quantum mechanical theory and just one interpretation of it, later called the Copenhagen interpretation. This theory was called indistinctly "quantum theory" or "quantum mechanics", given that there were no alternatives. Something similar still happens with the expression "classical mechanics" when it is used to refer to Newtonian mechanics (usually formulated in the Lagrange or the Hamilton mathematical formalisms). However, this use would have been equivocal by the end of the seventeenth century, when Cartesian mechanics was a serious rival theory of Newtonian mechanics. In principle, any branch or domain of physics can include several rival theories. To prevent equivocations and ambiguities, every theory of a given domain of physics should have a proper name that refers to it univocally. Nonetheless, we still lack a name for the first quantum theory, which is called simply "quantum mechanics" by most physicists.

There cannot be any doubt that there are different rival quantum theories. The issue of interpreting quantum mechanics then consists in interpreting *quantum mechanical theories* but not quantum physical theories generally. There is also a problem in interpreting other quantum theories, such as quantum field theory, but the problem could hardly be addressed without first elucidating the interpretative problem of quantum non-relativistic mechanics. In what follows I will deal exclusively with quantum non-relativistic mechanical theories (for short, I will refer to them as quantum theories)[14]. Given that we do not have a proper name for the first quantum theory, I will call it "the standard quantum theory", as I said before, because, besides being the first quantum theory, is the theory employed by most working scientists.

4. THEORIES VS. INTERPRETATIONS

As we have said, there are several different quantum theories among what are called interpretations of quantum mechanics. How many theories and which ones? Nobody can answer this question with certainty. This is part and parcel of the interpretative problem. In principle, any quantum theory is open to different interpretations, but how could we distinguish an interpretation of a given quantum theory from an alternative or rival theory? The problem is left unsolved in current literature because there is no agreement among scientists or philosophers of science on the identity of scientific theories. More precisely, we do not have a set of necessary and sufficient conditions for identifying theories. There are several accounts of theories and even skeptical stances about the possibility of finding such identity criterion[15]. However, we have at our disposal a partial criterion that has reached some degree of consensus among scientists and philosophers. All interpretations of a given theory must be *empirically equivalent* between them so that they must not imply new predictions. If a purported interpretation I_q of a quantum theory T_q permits the deduction of at least one prediction that is not implied by T_q, then, I_q is not an interpretation of T_q, but rather an alternative quantum theory. Auxiliary hypotheses are not relevant here, because the criterion can be extended to the conjunction of theories and auxiliaries. In this way, empirical equivalence provides a *necessary condition for the identity* of theories and non-empirical equivalence provides a *sufficient condition for the difference* of theories in general: (a) if T_1 and T_2 are not different theories but two different formulations of the same theory, they must be empirically equivalent; conversely, (b) if they are not empirically equivalent, they are not two formulations of the same theory but two different theories.

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[14] Laura Ruetsche, *Interpreting quantum theories*, New York, Oxford University Press, 2011 is one of the few books devoted to the interpretation of quantum field theories.

[15] Such as the one by Steven French, *There are no such things as theories*, Oxford, Oxford University Press, 2020.

This criterion was put forward by Van Fraassen as a necessary condition for all the interpretations of a given quantum theory[16]. It was also endorsed by some philosophically enlightened scientists, such as Ghirardi[17]. According to it, the so-called non-collapse theories, such as the GRW theory[18], are not interpretations of the SQT), but alternative theories. As is known, the GRW theory modifies the Schrödinger equation – adding stochastic and nonlinear terms to it- in such a way that it amounts to a different dynamics for the wave function. In practical situations, the predictions of the standard quantum theory and the GRW are very similar, so the difference between them is below the threshold of the sensitivity of our available measuring instruments. In the present situation, every experimental result that has confirmed the SQT also confirms the GRW theory. Nonetheless, in principle, although not yet in practice, it is possible to design a crucial experiment that could eventually confirm one of the two theories and disconfirm the other. This outcome can be generalized to all non-collapse theories, which should not be called interpretations of the quantum theory, as many scientists and philosophers still do. For the moment, all theories of this type are underdetermined by the available evidence, but this kind of underdetermination is merely transient, given the non-empirical equivalence between them and with the SQT.

The partial identity criterion, however, leaves open the question of whether some purported interpretations of quantum mechanics are alternative theories to the SQT. Bohm's hidden variables theory is an exemplary case. In 1952 Bohm put forward his theory as a "suggested interpretation of quantum theory in terms of 'hidden' variables". He regarded it as an alternative to the "usual interpretation" of the quantum theory[19], whereby in this expression he referred to what we know as the Copenhagen interpretation (which shows, by the way, that this name was not in use yet by the early 1950s). Along with his life, Bohm employed different names, such as "causal interpretation"[20] and "ontological interpretation"[21]. This might suggest that Bohm never intended to build an alternative or rival theory to the SQT. However, he also called his interpretation a "theory" and a "model"[22]. This ambiguity has been persistent until nowadays. The question of whether Bohm's account of quantum mechanics is an interpretation of the SQT or an alternative quantum mechanical theory has not been clarified yet. Different authors call it one

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[16] Bas C. van Fraassen, *Quantum mechanics: An empiricist view*, Oxford, Clarendon Press, 1991, p. 244.

[17] G. Ghirardi, *Sneaking a look at God's cards*, p. 377.

[18] G. Ghirardi, A. Rimini, T. Weber, "Unified dynamics for microscopic and macroscopic systems", in *Physical Review D*, vol. 34, nr. 2, 1986, pp. 470–491.

[19] David Bohm, "A suggested interpretation of quantum theory in terms of 'hidden variables'", p. 166.

[20] David Bohm, *Causality and chance in modern physics*, London, Routledge, 1957, p. 116.

[21] David Bohm, Basil Hiley, *The undivided universe: An ontological interpretation of quantum theory*, London, Routledge, 1993, p. 2.

[22] David Bohm, *Causality and chance in modern physics*, pp. 118–119.

way or another –or even both ways- in different contexts. The recent denomination of Bohmian mechanics does not add anything new to solve the question[23].

The reason for this predicament is that Bohm's account is, as far as we know, empirically equivalent to the SQT. And, for many scientists and philosophers of an empiricist persuasion, if two theories are empirically equivalent, they are not actually different but rather two different formulations of the same theory. Even when the two theories are not logically equivalent, and greatly differ in their ontologies, their empirical equivalence is regarded as a sufficient condition for their identity. Quine has expressed this stance with very clear terms:

> Theories can differ utterly in their objects, over which their variables of quantification range, and still be empirically equivalent […]. We hardly seem warranted in calling them two theories; they are two ways of expressing one and the same theory. It is interesting, then, that a theory can thus vary its ontology.[24]

Most positivist, empiricist, and verificationist philosophers will endorse the converse of criteria (a) and (b), that is, (c) if T_1 and T_2 are different theories, they are not empirically equivalent, and (d) if two theories are empirically equivalent, they are two formulations of the same theory. Criteria (a) and (b) seem to be acceptable for most philosophical positions. Instead, criteria (c) and (d) are not acceptable for realists, who should admit the possibility of empirically equivalent, but not logically or theoretically equivalent, theories. If we were two distinguish between empirically equivalent theories, we need a different criterion of identity. However, there is no agreement concerning such a criterion, as we will see.

5. INTERPRETATIONS VS. FORMULATIONS

We need also to distinguish the different interpretations of a given quantum theory from its different formulations. Any theory admits different formulations in different languages and mathematical formalisms. Axiomatized formal theories provide good examples. There are many axiomatic bases for Euclidean geometry (those of Hilbert, Poincaré, and Veblen, to mention just three of them). Each one provides a formulation of Euclidean geometry to the extent that implies the same theorems. The theory in question is determined by the class of its theorems and any theory that permits to prove exactly all those theorems is just a formulation of that

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[23] On Bohmian mechanics see Gérard Gouesbet, *Hidden worlds of quantum physics*, New York, Dover, 2013; Detlef Dürr, Stefan Teufel, *Bohmian mechanics: The physics and mathematics of quantum theory*, Berlin, Springer 2009; Peter J. Riggs, *Quantum causality: Conceptual issues in the causal theory of quantum mechanics*, Dordrecht, Springer, 2009. Most endorsers of Bohmian mechanics do not regard this theory as an interpretation of the SQT, but as a rival theory.

[24] W.V.O. Quine, *Pursuit of truth*, revised edition, Cambridge, MA, Harvard University Press, 1992, p. 96.

theory. In the case of axiomatized theories, it is sufficient to prove that their axiomatic bases are logically equivalent (that is, interreducible) to prove that they are formulations of the same theory. The criterion employed to identify a formulation of a given theory and to distinguish it from an alternative theory is just *logical equivalence*: (e) if two theories T_1 and T_2 are logically equivalent, they are two formulations of the same theory, and (f) if T_3 and T_4 are different theories, then they are not logically equivalent. Of course, the logical equivalence of theories implies their empirical equivalence, but the converse does not hold generally.

Physical theories can also be formulated in several different ways, through different, languages, mathematical formalisms, and axiomatic bases. The Newtonian, Lagrangian, and Hamiltonian formulations of classical mechanics are not regarded as three different theories but rather as three different formulations of the same theory. The reason is that the same results can be derived from each one and in all cases the calculations, although mathematically different, have the same outcomes. The SQT, in turn, has had Heisenberg's matrix formulation, Schrödinger's wavefunction formulation, and Feynman's path integral (or sum-over-histories formulation), among others. The choice of one formulation over the others is a purely pragmatic question based on non-factual criteria, such as simplicity or usefulness. Accordingly, it is perfectly acceptable to use one formulation to address one problem and another formulation to address another problem. The significant issues, however, are how to identify the theory in the first place, and then how to distinguish its different formulations from alternative theories and interpretations.

An interpretation of a quantum theory is not a reformulation of that theory. Consequently, an interpretation I_q of a quantum theory T_q cannot be logically equivalent to T_q. Otherwise, it is not an interpretation, but simply a reformulation of the theory expressed in a different language. Combining the results of this and the precedent section, we can establish two necessary conditions for any interpretation of a quantum theory. All interpretations of a given quantum theory T_q must be (i) empirically equivalent to T_q and (ii) non-logically equivalent to T_q. Van Fraassen has expressed these criteria in the following terms:

> Suppose we agree that there can, in logical principle, be more than one adequate interpretation of a theory. Then it follows at once that interpretations go beyond the theory; the theory + interpretation is logically stronger than the theory itself. (For how could there be differences between views, all of which accept the theory, unless they vary in what they add to it?). So an interpretation introduces factors not found in the theory originally – and what else does 'hidden variables' mean? The empirical superfluousness is required to ensure that no new or different predictions are forthcoming – else we have an alternative theory rather than an interpretation.[25]

These two criteria are just necessary conditions for an interpretation. Are they also sufficient? If not, what further criteria should an interpretation satisfy?

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[25] Bas C. van Fraassen, *Quantum mechanics: An empiricist view*, pp. 243–244.

The crucial point is that there are no clear-cut criteria to distinguish between quantum theories and interpretations. More generally, there is no consensus on the necessary and sufficient conditions to identify physical theories. Barrett has pointed out that there exist good reasons to be skeptical concerning the possibility of demarcating between quantum theories and their interpretations[26]. But in that case, all the interpretative enterprise is doomed to failure, or in the best case, to ambiguity. Barrett himself called Bohmian mechanics and the GRW theory "alternative formulations of quantum mechanics"[27], following an extended usage among physicists that do not distinguish between a physical theory, its different formulations, and its different interpretations. Daniel Styer, for instance, included Bohmian mechanics (under the alternative name of "pilot wave interpretation") among the formulations of quantum mechanics and claimed that the many-worlds interpretation "is close to the boundary between a 'formulation' and an 'interpretation'"[28]. However, according to the characterizations we have provided above, formulations and interpretations of physical theories are disjoint classes so that nothing can be at the same time a formulation and an interpretation of the same theory.

6. THE NO-INTERPRETATION STANCE

A possible response to the interpretative question is to dispose entirely of the interpretative problem. Some physicists have argued that the SQT does not need any interpretation. Fuchs and Peres have stated that position in the following terms:

> The thread common to all the nonstandard "interpretations" is the desire to create a new theory with features that correspond to some reality independent of our potential experiments. But, trying to fulfill a classical worldview by encumbering quantum mechanics with hidden variables, multiple worlds, consistency rules, or spontaneous collapse, without any improvement in its predictive power, only gives the illusion of a better understanding. Contrary to those desires, quantum theory does not describe physical reality. What it does provide is an algorithm for computing probabilities for the macroscopic events ("detector clicks") that are the consequences of our experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists.[29]

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[26] Jeffrey Barrett, *The conceptual foundations of quantum mechanics*, New York, Oxford University Press, 2019, pp. 49–50.

[27] *Ibidem*, p. 49.

[28] Daniel Styer *et al*, "Nine formulations of quantum mechanics", *American Journal of Physics*, 70, nr. 3, 2002, pp. 288–297, p. 295.

[29] Christopher Fuchs, Asher Peres, "Quantum theory needs no 'interpretation'", in *Physics Today*, 53, nr. 1, 2000, pp. 70–71, p. 70. Asher Peres, *Quantum theory: Concepts and methods*, New York, Kluwer, 2002 is a treatise written from this point of view.

There are at least two problems with these kinds of statements. First, a mathematical algorithm by itself does not permit the derivation of any prediction (either probabilistic or not) concerning the experimental results. A Hilbert space – the standard formalism of the SQT – does not predict by itself any detector clicks. The mathematical formalism of the theory must be given a *physical interpretation* to do that. Let us call it the *minimal interpretation*, that is, the interpretation necessary and sufficient to predict the occurrence of detector clicks or, more generally, experimental outcomes.

Second, this position uncritically assumes a philosophical stand concerning the aim of physical theories, or science generally, according to which the fundamental aim of a scientific theory is just predicting experimental outcomes. In this construal, internal (and perhaps, external) consistency and empirical adequacy are all we should demand from a theory in physics. This claim is common to all instrumentalist or antirealist stances towards quantum theories (such as quantum Bayesianism). For a realist, this stance is not acceptable considering the very aims she assigns to science. Prediction, nobody would deny it, is a respectable scientific aim, but scientific theories must also provide us with a literally (approximately) true description of the world, as well as an explanation of physical phenomena that permits us to understand them. A purely predictive mathematical algorithm will obviously fail at satisfying those further aims.

Besides the disagreement concerning the fundamental aims of physical theories in general, the Fuchs and Peres no-interpretation stance cannot dispose of the interpretative problem. To say that the SQT does not describe the physical and, therefore, that the wave function Ψ does not refer to anything in the real-world amounts to endorsing one interpretation among others.

Fuchs and Peres assume that there is just one quantum theory (namely the SQT) and that it needs no interpretation. On the other extreme side of the debate, we find the view according to which there are no interpretations of the SQT at all but rather several different theories. Tim Maudlin, for instance, has recently claimed that the SQT is not a theory, because it does not have an ontology, but just a recipe to extract empirical predictions. The recipe consists essentially of the following three steps: a) assign a wave function to a physical system; b) apply the Schrödinger equation to calculate the dynamical evolution of that wave function; and c) apply the Born rule to the former outcome to extract probabilistic predictions concerning experimental outcomes. Notice that we do not need the Hilbert space formalism to apply this recipe.

According to Maudlin, the so-called interpretations of quantum mechanics are not interpretations but rather genuine rival theories. In his words:

> A physical theory should contain a physical *ontology*: What the theory postulates to exist as physically real. And it should also contain *dynamics*: laws (either deterministic or probabilistic) describing how these physically real entities behave. In a precise physical theory, both the ontology and the dynamics are represented in sharp mathematical terms. But it is exactly in this

> sense that the quantum-mechanical prediction-making recipe is not a physical theory. It does not specify what physically exists and how it behaves, but rather gives a (slightly vague) procedure for making statistical predictions about the outcomes of experiments. And what are often called "alternative interpretations of quantum theory" are rather alternative precise physical theories with exactly defined physical ontologies and dynamics that (if true) would explain why the quantum recipe works as well as it does.[30]

I will address later the issue concerning the ontology of theories. For the moment, two remarks will suffice to make Maudlin's stance scarcely appealing. First, most scientists and philosophers regard the SQT as a genuine physical theory, and even as the only quantum theory. Second, many interpretations of the SQT do not provide an ontology for that theory, for instance, all antirealist or epistemic interpretations, from The Copenhagen interpretation to quantum Bayesianism. According to Maudlin criteria for a physical theory, they qualify neither as legitimate quantum theories nor as interpretations of any quantum theory[31]. This stance is at odds both with scientific practice and most philosophies of physics.

7. WHAT AN INTERPRETATION IS NOT

Most scientists do not distinguish between formulations or reformulations of theories, interpretations of that theories, and alternative theories. Usually, they regard as interpretations of the SQT some things that are clearly different theories. Some philosophers have endorsed this broad (and vague) use of the concept of interpretation. As Peter Lewis writes:

> Note that I am using "interpreting" in a broad sense, to include supplementing or even replacing quantum mechanics as a physical theory. Some may wish to divide "genuine interpretations" – those that work with quantum mechanics as it stands- from more revisionary projects. But I will stick with broad usage, which has become somewhat canonical in the literature.[32]

It is not clear what "supplementing" the SQT amounts to if that is not a change of theory. In any case, replacing a given quantum theory with another quantum theory should not count as an interpretation of the first theory. It seems clear enough, for instance, that replacing Newtonian mechanics with special

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[30] Tim Maudlin, *Philosophy of physics: Quantum mechanics*, Princeton, Princeton University Press, 2019, pp. 4–5.

[31] It is no wonder that Maudlin neglects the Copenhagen interpretation. In his view, the quantum theories at stake are Bohmian mechanics, many worlds, and GRW, of which he favors the first one.

[32] Peter Lewis, *Quantum ontology: A Guide to the metaphysics of quantum mechanics*, New York, Oxford University Press, 2016, p. 184.

relativity is not an interpretation of Newtonian mechanics in any reasonable sense of the word. In what follows I will aim at delimitating a *narrow sense* of “interpretation” that permits us to distinguish the interpretations of a given quantum theory from its reformulations and its alternative rival theories. Before attempting such demarcation, we should be clear about what an interpretation of a quantum theory is not.

First, interpreting a quantum theory does not consist in *assigning a physical meaning* to an uninterpreted mathematical formalism. That formalism must be already interpreted, in some way or another; otherwise, the theory is purely formal and not a physical theory at all. A physical meaning must be assigned to the symbols of the mathematical formalism of the SQT if that theory is to be used to predict experimental outcomes. Second, interpreting a quantum theory does not consist in *axiomatizing* that theory or *reformulating* it in any way. As we have said, a given theory admits many different formulations (that is, logically equivalent presentations) in different languages and formalisms. Some formulations may be more clear or more useful than others for different purposes, either for calculation, application, or teaching. However, reformulating a theory should not be regarded as providing an interpretation of it, in the specific and narrow sense in which we speak of interpretations of the SQT. Axiomatizing or reformulating the SQT are useful enterprises in themselves, but they are not part of the interpretative task. Third, interpreting a quantum theory does not consist in building an *alternative* or *rival theory*. If we identify a theory as a different theory from the SQT, that theory is not an interpretation of the SQT.

8. IDENTIFYING THEORIES

The first step in providing an interpretation of a theory is to identify that theory. This looks like a truism but in practice is not an easy task at all. The reason is that we do not have at our disposal a set of criteria based on necessary and sufficient identity conditions for theories. Working scientists do not even address this problem and philosophers of science have not reached an agreement on the issue. This is not the place to review this broad topic. The mainstream among philosophers of science in the last decades, at least from the 1980s, has been to conceive of empirical theories as collections of related models. This model-theoretic approach has evolved in several partially different accounts[33]. For the sake of the argument, I will employ hereafter the classical conception of theories as sets of

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[33] The standard name is still “the semantic view of theories”, but “the model-theoretic view” is a more accurate tag. For general accounts of the different model-theoretic approaches to empirical theories, see Steven French, *There are no such things as theories*, Oxford, Oxford University Press, 2020; and Roman Frigg, *Models and theories: A philosophical inquiry*, London, Routledge, 2023. Hans Halvorson, *The logic in philosophy of science*, Cambridge, Cambridge University Press, 2019 provides a detailed study of the logical structure of theories, including the semantic view.

propositions closed under logical consequence. When a theory is axiomatized, its different formulations are easily distinguishable by their axiomatic bases. The theory is then determined by the set of its theorems, not by a given axiomatic basis written in some language. All the sets of axioms that are logically equivalent are different formulations of the same theory. This strategy is not available for quantum theories because they are not axiomatized. We then must use a more general concept of equivalence, according to which two theories are logically equivalent if they have the same logical consequences (that is, T_1 and T_2 are equivalent if and only if $Cn(T_1) = Cn(T_2)$). In that case, we say that T_1 and T_2 are two different formulations of the same theory. From a logical point of view, a theory can be defined as the equivalence class of all its formulations, as Quine suggested[34]. This move, however, leaves open the problem of distinguishing between the different formulations of a given theory and alternative empirically equivalent theories.

Concerning physical theories, the first thing to be clarified is the distinction between a given physical theory and its mathematical formalism. A physical theory is not a mathematical formalism and cannot be identified solely through its mathematical formalism. On the one hand, the same theory admits different formulations that use different mathematical formalisms; on the other hand, the same mathematical formalism can be used to formulate different rival theories. For instance, many alternatives to classical mechanics can be formulated using the Lagrangian or the Hamiltonian formalisms, and many alternatives to the SQT can be formulated using the Hilbert space formalism. Besides, a physical theory can be formulated without using any specific mathematical formalism, although modern physics has become essentially mathematical since the seventeenth century.

More importantly, a mathematical formalism by itself lacks empirical content and cannot be used to predict the occurrence of any physical event (not even probabilistically). A Hilbert space by itself is as devoid of physical content as the differential calculus or the arithmetic of natural numbers. Sometimes there is a preferred formalism that becomes standard for a given theory, as is the case with the Hilbert space in the formulation of the SQT. The choice of a definite formalism is essentially conventional and based on pragmatic reasons. Once a standard formalism is selected for the formulation of a physical theory, the physical interpretation of some of its mathematical symbols must be fixed; otherwise, the theory cannot be used to make empirical predictions. There cannot be disagreement concerning the physical meaning of that mathematical formalism, otherwise the interpretative issue will become purely equivocal. A mathematical formalism and its minimal interpretation are both necessary to any physical theory. For that reason, it is indispensable to identify which are the mathematical symbols to which a physical interpretation is assigned. I will turn to this point next.

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[34] W.V.O. Quine, “On empirically equivalent systems of the world”, in *Erkenntnis*, vol. 9, nr. 2, 1975, pp. 313–328, p. 321.

9. IDENTIFYING THE STANDARD QUANTUM THEORY

Among the different formulations of the SQT, one has become the standard for learning and teaching, as I said, the Dirac-Von Neumann formulation employing the Hilbert space formalism. This formalism has a widely accepted minimal interpretation that assigns a physical meaning to some mathematical symbols of Hilbert spaces, such as vectors and operators. The primitive physical concepts employed by the minimal interpretation are those of *physical system*, *state* (of a physical system), and *observable*. Other concepts, such as those of *simple* and *composite system*s or *pure* and *mixed states*, are defined through the primitives.

The usual formulation of the SQT employs, either as explicit postulates or as implicit assumptions, a whole set of semantic rules for interpreting the different elements of the Hilbert space formalism. For instance, it is usual to claim (as in Weinberg, 2015: 52 and 61) that the first two postulates of the SQT are the following:

P_1: A Normalized vector in a Hilbert space represents the state of a physical system.

P_2: Hermitian operators on a Hilbert space represent observables (measurable physical quantities, such as position, momentum, or energy).

From a logical point of view, however, the first postulate should be the following:

P_3: A Hilbert space $\mathcal{H}$ represents a physical system.

It seems evident that postulates P_1-P_3 lack empirical content to the extent that they do not make any assertion about physical phenomena. For that reason, they are not capable of being tested by experience, that is, they cannot be confirmed or refuted by any evidence. They simply assign physical meaning to some mathematical symbols belonging to the formalism of a given formulation of the SQT. Weinberg rightly called them “interpretive postulates”[35]. There are many other semantic rules employed to that aim. Most are not stated as postulates, but rather given as implicit or explicit rules for assigning a physical meaning to other symbols of the Hilbert space formalism. For instance, the so-called composition rule is sometimes included among the postulates:

P_4: The tensor product of two vectors belonging to two Hilbert spaces represents a composite physical system. More precisely, if a vector $|V\rangle$ of a Hilbert space $\mathcal{H}_1$ represents a physical system S_1 and a vector $|W\rangle$ of a Hilbert space $\mathcal{H}_2$ represents a physical system S_2, then, the tensor product $|V\rangle \otimes |W\rangle$ of $\mathcal{H}_1 \otimes \mathcal{H}_2$ represents the composite physical system $S_1 + S_2$.

This, again, is an interpretive rule devoid of any empirical meaning and useless in any formulation of the SQT that does not use the Hilbert space formalism.

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[35] Steven Weinberg, *Lectures on quantum mechanics*, p. 61.

The so-called superposition principle is often included among the postulates of the SQT:

P_5: If two vectors $|V\rangle$ and $|W\rangle$ of a Hilbert space $\mathcal{H}$ represent two possible states of a physical system *S*, then, the linear combination $\alpha|V\rangle + \beta|W\rangle$ (where α and β are two complex numbers such that $\alpha^2 + \beta^2 = 1$) is a vector that represents a possible state of *S*.

There are other interpretative rules for the SQT when formulated in the standard Hilbert space formalism. I do not intend to offer here a precise or complete formulation of these postulates, but, under some formulation or another, they are used as postulates of the SQT in most textbooks[36] .

Three remarks on the above postulates are in order here. First, the concepts of *particle*, *wave*, and *field* do not belong to the minimal interpretation of the SQT. All the postulates of this theory can be rigorously formulated without employing these concepts. They are concepts belonging to classical physics that are difficult to reconcile with quantum physics. No sound formulation of the SQT states that it is a theory that refers primarily to particles, waves, or fields (despite the misnomer "wave function").[37] Second, the above postulates are not statements of laws of nature. They are simply *semantic rules* that assign physical meaning to the mathematical symbols of the Hilbert space formalism. They are *conventions*, not empirical hypotheses. Third, none of the postulates is essential for the SQT. What I mean is that they are *relative to a given formulation* of the theory, but they can be useless within another formulation (namely, one that does not employ the Hilbert space formalism). The Hilbert space formalism must be conceived of as just a useful, but in principle dispensable, tool for the formulation of the SQT. However, nothing essential of the SQT, *as a physical theory*, is dependent upon that formalism.

The above postulates P_1-P_5 do not express by themselves any physical theory because they refer to the mathematical symbols of the Hilbert space formalism but not to physical systems. The specific physical content of the SQT is given (remember, in the Dirac-Von Neumann formulation) by the two dynamical laws for the evolution of physical systems:

P_6: The linear dynamical law: if no measurement is performed on a physical system *S*, it will evolve in time according to the Schrödinger equation.

P_7: The non-linear dynamical law: if a measurement is performed on a physical system *S*, it will instantaneously and randomly jump to an eigenstate of the observable being measured.

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[36] For a more complete list of eleven interpretative rules for the SQT see Jeffrey Barrett, *The conceptual foundations of quantum mechanics*, p. 41.

[37] The expression "the wave function" predates the formulation of the SQT. In the framework of the Hilbert space formalism, it should be called "the state vector" and be written as $|\Psi\rangle$. Both expressions are still in use, although they are not strictly synonymous.

These two postulates are the expression of the double dynamics of quantum systems: the unitary dynamics, given by the Schrödinger law, and the collapse dynamics (usually called "collapse of the wave function" or "reduction of the state vector"), given by the non-linear or collapse law of evolution. As is well known, those postulates are hard to reconcile and together originate the traditional problem of measurement, perhaps the main conceptual problem of the SQT, although by no means the only one. The two laws of evolution are independent of the Hilbert space formalism and, consequently, can be used in any formulation of the SQT (perhaps with some changes in their statements). They are not semantical rules but empirical hypotheses; the expressions of two laws of nature, such as Newton's or Maxwell's laws, or any other law of physics. Of course, the fact that they refer explicitly to measurements makes them very special laws, in this respect, with no analog in other physical laws. The very fact of having two different (and mutually incompatible) laws for the evolution of physical systems also makes the SQT unique among physical theories[38].

I do not intend here a complete formulation of the SQT. Nonetheless, if postulates P_6 and P_7 (or any equivalents) are regarded as *necessary* to identify the SQT, any non-trivial change in these postulates amounts to a change of theory. Notice that changing one of these postulates also implies a change of theory from the point of view of the model-theoretic view of theories, because it changes the class of models of that theory, not just the set of its theorems. This is a standard criterion to identify theories, so that, from this point of view, those interpretations that change the dynamics, by modifying Schrödinger's equation, qualify as alternative theories to the SQT, as is the case with the GRW theory. By the same token, removing one of these postulates implies a change in the SQT, as is the case with the many-worlds interpretation, at least with the so-called *pure wave mechanics*, which is obtained by dropping the collapse postulate. In turn, if Bohmian mechanics is regarded as an *extension* of the SQT[39], it then turns out to be an alternative theory to the SQT. The extension in this case consists of supplementing the linear dynamics of the SQT with the guiding equation for the particles. However, if Bohmian mechanics is regarded as a species of non-collapse theory, it cannot be consistent with the non-linear dynamics of the SQT. It then must not be conceived as an extension of the SQT but rather as a revision. In any case, it turns out to be an alternative theory of the SQT.

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[38] The two laws are incompatible in the sense that a given physical system cannot evolve linearly and non-linearly at the same time. When and why a system ends its linear evolution and collapses is nothing but the problem of measurement.

[39] See, for instance, Jeffrey Bub, *Bananaworld: Quantum mechanics for primates*, Oxford, Oxford University Press, 2016, p. 44.

All the preceding considerations depend upon the identification of the SQT through a set of postulates. This strategy can be questioned, because there is no agreement concerning an axiomatic formulation of this theory. Nonetheless, there cannot be doubt that the linear and non-linear dynamical laws are two essential postulates of the SQT. In any event, we do not have at our disposal a complete set of postulates necessary and sufficient to identify the SQT. This predicament can promote some skeptical attitudes towards the identity of theories. Steven French has expressed skepticism concerning identity criteria and endorsed an eliminativist stance toward theories. According to the very title of his book, *there are no such things as theories*. Quantum mechanics is precisely French's paradigmatic example of a no-theory.

> We need to abandon the idea that the history of the field, or the relevant practices of the scientists in general, supports the claim that there is 'a', or 'the' theory of quantum mechanics, as a unitary and well-delineated entity, with definite identity conditions. This was clearly not the case at the time of the so-called quantum revolution, nor in the immediate aftermath, nor subsequently, if we understand a theory, qua entity, as incorporating some claim as to how the world is or could be.[40]

Another skeptical stance consists in regarding theories as essentially vague entities without well-defined identity conditions. The distinguished physicist Robert Geroch has expressed this view in these terms:

> […] theories consist of an enormous number of ideas, arguments, hunches, vague feelings, value judgments, and so on, all arranged in a maze. These various ingredients are connected in a complicated way. It is this entire body of material that is 'the theory'. One's mental picture of the theory is this nebulous mass taken as a whole.[41]

If theories are vague entities, the distinction between formulations, interpretations, and alternatives to a given theory cannot even be drawn. Consequently, the problem of identifying the different interpretations of the same theory is doomed to failure. We then must assume that physical theories have identity conditions (although we can disagree about which are exactly those conditions). In particular, we have to assume that the SQT can be identified otherwise, it cannot be distinguished from alternative theories, and, above all, no interpretation of it could be produced.

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[40] Steven French, *There are no such things as theories*, pp. 208–209.

[41] Robert Geroch, *General relativity from A to B*, Chicago, The University of Chicago Press, 1978, p. 183.

10. INTERPRETATION AS ONTOLOGY

The minimal interpretation of the SQT (which certainly includes more than the postulates *P1-P5*) gives a physical interpretation to the mathematical symbols of the Hilbert space formalism. To do that, it employs some primitive concepts – such as those of *physical system*, *state*, and *observable*-, which are not defined or explained in any way. Usually, they are introduced by means of *examples* (for instance, a particle moving inside an empty box and others). For that reason, the minimal interpretation does not provide a full-blooded or robust ontology, and not even a primitive ontology, for the SQT. For example, it does not specify what are the physical systems to which the theory intends to refer, or what kinds of physical systems exist in the world according to that theory. Above all, it does not say that they are particles (elementary or not), waves, or fields. Assuming there are particles and/or fields in the real world goes beyond the minimal interpretation. Moreover, this assumption is not necessary to derive from the SQT predictions concerning experimental outcomes. Finally, the minimal interpretation does not fix the complete meaning or the reference of the wave function, so that it leaves room for different accounts of it, specially, for different ontologies.

According to an increasing number of philosophers of science, as well as for some scientists, interpreting a quantum theory consists in providing an ontology for that theory. Ven Fraassen, for instance, states the problem by saying that an interpretation of the SQT aims at answering the question of "how could the world possibly be way this theory says it is?"[42] In tur, Dean Rickles claims that "interpretation is closely linked to ontology: to interpret is often just to provide an ontology"[43]. We can distinguish three different approaches to the ontological question, which I will label: a) one theory-one ontology; b) many theories-many ontologies; and c) one theory-many ontologies. I will address them in that order.

According to the one theory-one ontology approach, interpreting a quantum theory amounts to finding *the unique* ontology of that theory. John Norton, for instance, has claimed that "*interpretations of quantum theory* is the general term given to the problem of identifying the physical ontology associated with the mathematical structures of quantum theory"[44]. This way of posing a problem seems to assume that there is a one-to-one correspondence between mathematical formalisms and ontologies, so that, given a mathematical formalism for the SQT (the Hilbert space formalism), there exist just one ontology compatible with that formalism. But this is a wrong assumption. In the first place, mathematical formalisms are ontologically neutral, and, for that reason, we can never find the ontology of the SQT by scrutinizing the Hilbert space formalism. This formalism is

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[42] Bas C. van Fraassen, *Quantum mechanics*, p. 242.

[43] Dean Rickles, *The philosophy of physics*, Cambridge, Polity Press, 2016, p. 10.

[44] John Norton, "The three principal problems of philosophy of modern physics", https://sites.pitt.edu/~jdnorton/Goodies/three_problems/index.html, accessed: 30 June 2023.

as neutral as the differential calculus. It is just a tool that can be used to formulate many different (non-empirically equivalent) quantum theories. In the second place, the same theory can be formulated using different mathematical formalism. And this will imply that there are different ontologies for the same theory. In the third place, a given formulation of a theory (and we always deal with theory formulations) is compatible with many different ontologies (as I will argue soon).

The many theories-many ontologies approach is the view according to which there are no interpretations of quantum theories, but rather different rival quantum theories[45]. On this view, every physical theory must specify an ontology, otherwise, it is not a theory. For that reason, the SQT is not a theory but a mere recipe for making calculations and predictions. *A fortiori*, there is not such a thing as the Copenhagen interpretation of the SQT. The main rival quantum theories are then Bohmian Mechanics, Many-worlds, and GRW. Other purported interpretations are disregarded as no-theories at all. This approach is committed to a strong realist construal of physical theories. The link seems to be so strong that a physical theory cannot admit an anti-realist interpretation. However, this is not compatible with scientific practice (where many scientists are instrumentalists) or with the present state of the philosophy of science (where many philosophers are not realists). Of course, this approach could be interpreted as a normative stance concerning physical theories but, in that case, it needs a further justification[46].

The main problem with this approach does not lie in its realist stance but rather in the fact that it leads us to multiply the number of quantum theories at stake without a limit. Several different ontologies have been found for the same quantum theory, so they must be regarded as different theories. For instance, we now know three different ontologies for the GRW theory: the *wave function realism* (according to which the wave function refers to a multi-dimensional field in configuration space), the *mass density* ontology (according to which the wave function refers to a continuous distribution of matter in ordinary three-dimensional space), and the *flash* ontology (according to which the wave function refers to short-duration events called flashes that occur in four-dimensional spacetime). Those are clearly incompatible ontologies. Consequently, on this view, GRW(r), GWR(m), and GWR(f) must be regarded as three different theories, although their formalisms, their equations, and their postulates are identical[47].

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[45] This view is endorsed, among others, by Travis Norsen, *Foundations of quantum mechanics*, Cham, Springer, 2017; Tim Maudlin, *Philosophy of physics*, Princeton, Princeton University Press, 2019; and Detlef Dürr, Dustin Lazarovici, *Understanding quantum mechanics*, Cham, Springer, 2020.

[46] David Wallace, *The emergent multiverse: Quantum theory according to the Everett interpretation*, Oxford, Oxford University Press, 2012, chapter 1, has argued in favor of such strong realist construal of physical theories. I do not intend to discuss his arguments here, which are not specific to quantum theories.

[47] Those ontologies are described with more detail in Travis Norsen, *Foundations of quantum mechanics*, chapter 9; Jeffrey Barrett, *The conceptual foundations of quantum mechanics*, chapter 8; Tim Maudlin, *Philosophy of physics*, chapter 4; and Detlef Dürr, Dustin Lazarovici, *Understanding quantum mechanics*, chapter 5.

The predicament is even worst for the so-called Many-worlds theory, the pure wave dynamics of the quantum theory (according to which the wave function always evolves linearly and never collapses) can be supplemented with many ontologies. According to Barrett:

> Pure wave mechanics may be reformulated in terms of a single world with illusions of determinate records, a single world with relative facts, splitting worlds, decohering worlds, a single world with one mind for each observer, a single world with many minds for each observer, many threads, many maps, or physical hidden variables.[48]

In order to be coherent with the many theories-many ontologies approach, we should say that instead of the single Many-worlds theory we have nine different empirically equivalent theories[49]. And surely many more in the future.

The one theory-many ontologies approach claims that any quantum theory, in any formulation, is compatible with different ontologies. And this is also true of all physical theories. Thus, there is no such thing as the ontology of the SQT (or any other quantum theory). If we pose the interpretative problem in terms of the possible worlds of which the SQT is true, the answer is that the SQT is compatible with many different worlds. There are many possible worlds compatible with the SQT (or with other quantum theories), although not every possible world is compatible with it. The postulates of the SQT constrain the class of the possible worlds compatible with the theory (in a similar way in which the axioms of a formal theory constrain the class of the models of that theory), but they do not select one of those possible worlds as the real world. This implies that there is no such thing as "the ontology of SQT". Therefore, the ontology we select for a quantum theory, if any, cannot be any more than an *intended ontology* (again, in analogy with the intended model of a formal theory).

It is true that the minimal interpretation of the SQT provides some ontological tenets for that theory, for instance, it states that there are physical systems in the world, that those systems are endowed with measurable properties, and that they can be in some quantum states in which some of their properties (but not all of them) have definite values when they are measured. However, this is just a *thin ontology* that does not provide a robust ontology, i. e., a full-blown picture of the world according to the SQT, which is precisely what we want to obtain from an interpretation.

The minimal interpretation of the SQT does not fix the full meaning (either the sense or the reference) of the wave function, its main mathematical object that is supposed to represent the quantum state of a physical system. Physicists often call wave function both to the state vector $|\psi\rangle$, which is a mathematical object, as

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[48] Jeffrey Barrett, *The conceptual foundations of quantum mechanics*, p. 220.

[49] These different ontologies are too numerous to be described here. See Jeffrey Barrett, *op. cit.*, chapters 9 and 10, for a detailed account of each one.

its name indicates, and to the physical entity to which this vector supposedly refers (in case it does it). To prevent equivocation, it is better to avoid the expression "wave function" and, following Maudlin[50], to call "state vector" to the mathematical object of the formalism of the SQT and "quantum state" to the features of physical systems that this vector represents. The main task of any interpretation of the SQT that intends to complete its minimal interpretation is to specify the meaning of the state vector, especially to what kind of entity it refers to. Most interpretations can be classified into two main groups: the *ontic* interpretations, according to which the state vector refers to a single entity or to the state of individual physical systems, and the *epistemic* interpretations, according to which the state vector does not refer to any single entity or state of individual systems. In turn, epistemic interpretations can be divided into two groups: *statistical* interpretations, which claim that the state vector refers to collectives or ensembles of individual physical systems, describing properties or features of those ensembles; and *credal* (sometimes called *subjective* or simply epistemic) interpretations, according to which the state vector does not refer to anything at all in the physical world, but rather to the knowledge or beliefs of individual human agents[51]. In this view, the quantum probabilities are regarded as subjective degrees of belief, as is the case within the orthodox Bayesian statistics[52].

There exist several interpretations within each group, which are too numerous to be reviewed here. For example, among the ontic interpretations, the so-called *wave function realism* claims that the state vector refers to a multidimensional field in 3*N*- configuration space[53]. This statement must be construed as the claim that each particular state vector refers to the different states of this hyperfield, given that the state vector in SQT does not represent a physical system (this is represented by a Hilbert space) but the state of a physical system. According to a different ontic interpretation, the state vector refers to the *quantum state* of individual physical systems in ordinary space, such as particles or stars, assuming

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[50] Tim Maudlin, *Philosophy of physics*, p. 37.

[51] This terminology comes from Tim Maudlin, *Philosophy of physics*, pp. 79–81, but nowadays have become the standard one.

[52] See, for instance, Jeffrey Bub, *Interpreting the quantum world*, Cambridge, Cambridge University Press, 1997. Richard Healey, *The quantum revolution in philosophy*, New York, Oxford University Press, 2017, p. 247, has summarized the credal interpretation in crystal clear terms: "quantum theory itself describes the world neither as deterministic nor as indeterministic, since it does not describe the world at all. When correctly applied, quantum theory offers advice on how to set our degrees of belief in matters about which we are not currently in a position to be certain."

[53] This interpretation was put forward by David Albert, "Elementary quantum metaphysics", in J. Cushing, A. Fine, S. Goldstein, (eds.), *Bohmian mechanics and quantum theory: An appraisal*, Dordrecht, Kluwer, 1996, pp. 277–284, and extensively discussed by different authors in Alyssa Ney & David Albert (eds.), *The wave function: Essays on the metaphysics of quantum mechanics*, New York, Oxford University Press, 2013. A sustained defense of this interpretation can be found in Alyssa Ney, *The world in the wave function. A metaphysics for quantum physics*, New York, Oxford University Press, 2021. Notice that it can be applied to any quantum theory that uses a wave function defined on configuration space.

that the SQT is a universal theory that applies to any physical system (actually, no postulate of the theory limits its domain of application to the microworld). Again, if we accept that a physical theory must have a single unique ontology, these two interpretations turn out to be two different theories, even though they do not introduce any change in the formalism or the minimal interpretation of the SQT.

11. THE ONTOLOGICAL MUDDLE

As we have seen in the preceding section, it is possible to conceive of different ontologies for the same physical theory. If so, on which criteria should we choose an intended ontology for a quantum theory? This is a serious problem for any kind of ontological pluralism. Once we have acknowledged the possibility of several ontologies for the same theory, we have blocked the way from the theory to the real world. We then cannot say that our preferred ontology is the ontology of the real world. If we grant all this, we will find the following predicament: in the first place, we cannot ascertain which ontology is the ontology of the real world (because any experiment that confirms the SQT theory will also confirm this theory with any intended ontology); in the second, place, we do not have any decisive criteria to select an intended ontology.

Certainly, everybody will agree on the fact that the choice between different ontologies should not be a matter of personal taste. I will add that it cannot be based on purely aesthetic criteria either, on which it is hard to obtain consensus. Experience cannot decide the issue, given that all interpretations of the same theory are empirically equivalent. Non-factual criteria, such as simplicity, generality, and others of the like, are best suited for the choice of theories (above all, when they are underdetermined by the available evidence) but their application to rival ontologies is dubious. Ontological parsimony (a version of Ockham's razor) could be invoked, but it is difficult to apply, and it might leave the issue in an undecided state. Just think about the choice between a three-dimensional and a 3*N*-dimensional spatial ontology for the state vector. Which one is more parsimonious? Is there any advantage in assuming that there is just one physical entity, the hyperfield, endowed with a possibly infinite number of dimensions over the assumption that there is a possible infinite number of physical entities (say, particles) that move in the three-dimensional ordinary space? I do not have the answer, but I think there are grounds for reasonable doubts that can drive us to a skeptical attitude concerning the choice of a definite ontology.

We certainly are not committed to accept that every mathematical feature of the state vector represents some real physical entity or feature of the quantum state. Therefore, a 3*N*-dimensional ontology is not forced on us by the very fact that the state vector is a mathematical object defined in configuration space. We can think of the configuration space as a mere mathematical tool without physical counterparts, in the same way as we think of the phase space in classical physics. But if we adopt this stance, we can also refuse to accept that other mathematical degrees of freedom of the state vector correspond to real degrees of freedom in the quantum state, or in physical systems generally. Maudlin, for instance, has claimed that the overall phase of the state vector does not represent any degree of freedom of the quantum state, given that no prediction of the SQT is changed if we multiply a state vector by an arbitrary overall phase[54]. In any case, it is not clear which are the required criteria to decide this issue. Should we adopt the stance that the superposition of different state vectors does not represent any real superposition of quantum states? Is the state superposition just an artifact of the formalism of quantum theories without any counterpart in the physical world?

From a practical or pragmatic point of view, the one adopted by most laboratory physicists, we can dispense entirely with an intended robust ontology for quantum theories, beyond the thin ontology provided by the minimal interpretation of their mathematical formalisms. On the one hand, the successful use of the SQT shows that this stance is not only possible, but also feasible. On the second hand, as the history of science shows, some physical theories do not have even an intended ontology, contrary to what Maudlin, Wallace, and other realist philosophers of science think[55]. The theory of special relativity provides a good example. This theory is compatible with a pure field ontology, without particles (Einstein’s preferred ontology), with a pure particle ontology, where the particles act (non-instantaneously) at a distance without intermediate fields, and with a dualistic ontology of particles and fields. Moreover, it is compatible with a pure event ontology and with an ontology of objects and properties. Finally, it does not commit us to a four-dimensional ontology, given that it can be formulated without resourcing to spacetime (as Einstein did it).

Whatever the intended ontology for a quantum theory we provide, we will be unable to recover a common-sense ontology, according to which there are spatially separated objects that possess all their properties in act. As far as we know, every consistent and empirically adequate quantum theory must be at the same time *non-local and contextual*, as is the case, for instance, with the SQT and Bohmian mechanics. If this is true, we will also be unable to recover a local and non-contextual ontology, as is the one traditionally assigned to classical field theories. That was certainly Einstein’s dream, but until now it has revealed to be impossible. In the last resort, quantum theories will force us to decide among unfamiliar purported ontologies, and, for the moment, we do not have at our disposal any decisive criteria to prefer one over the others.

[54] Tim Maudlin, *Philosophy of physics*, p. 90.

[55] See, for instance, David Wallace’s, *The emergent multiverse*, p. 12, remarks on classical fields. Are they substances or just properties of spacetime points? Both ontologies are compatible with field theories, in my view.

12. CONCLUSIONS

I have tried to shed some light on the issue of distinguishing between interpretations and rival theories of the SQT. On the one hand, I have proposed two partial criteria to distinguish between different formulations of the same theory, different interpretations of a theory, and rival theories to it. According to the first criterion, based on empirical equivalence, all interpretations of a quantum theory must be empirically equivalent to it otherwise, they are rival theories, and must not be logically equivalent to it otherwise, they are different formulations of such theory. Based on this criterion, GRW and all spontaneous collapse theories are not interpretations of the SQT, but alternative rival theories. On this criterion, Bohmian mechanics and Everett's interpretation do not qualify as alternative theories. According to the second criterion, on which we identify a theory through its fundamental postulates, Bohmian mechanics and Everett's interpretation are rival theories of the SQT. This outcome depends upon the assumption that the collapse postulate is essential to the SQT. In any case, it seems evident that Bohmian mechanics introduces changes into the SQT, both by supplementing the space of the states with the so-called hidden variables (essentially, the particles' positions) and by adding a new equation for the motion of the material particles. In turn, Everett's interpretation drops the collapse postulate, without changing the Schrödinger equation, adding new dynamical equations, or supplementing the space of the states with new variables. For this reason, it has been regarded as a pure interpretation of the SQT. However, it seems reasonable to view both interpretations as rival theories regarding the SQT.

The different interpretations of the SQT must show some theoretical differences between them, that is, they must not be theoretically equivalent. I have not intended to elucidate here the concept of theoretical equivalence, which has been very much discussed by philosophers of science[56]. An interpretation of the SQT must add some theoretical structure to the minimal interpretation of that theory otherwise they would be logically equivalent to it. This holds generally for any interpretation of any quantum theory. A purported interpretation of a quantum theory must be empirically equivalent to it, but not theoretically equivalent. Thus, given that no possible experience could discriminate between different interpretations of a quantum theory, the choice must be based on non-factual criteria, as is the case with empirically equivalent theories.

Many philosophers have thought that the extra structure added to the minimal interpretation of a quantum theory is a full-blooded ontology for that theory. Accordingly, every interpretation must say what the theory is about and how the world is according to the theory in question. This is a rather naïve – or naïve

[56] On this issue, see Hans Halvorson, *The logic in philosophy of science*, Cambridge, Cambridge University Press, 2019, and the references cited there.

realist – view of the ontology. As I have argued, there is no such thing as the unique ontology of a physical theory. In the first place, the mathematical formalism is ontologically neutral. In the second place, any quantum theory minimally interpreted is compatible with different ontologies. We can specify an ontology for a quantum theory, but that cannot be more than an intended ontology. As I have argued, we do not have any decisive criteria to choose one ontology among others. If interpretating a quantum theory consists in assigning an ontology to that theory, the conflict of interpretations is not only unavoidable, but also undecidable.