LOGICAL SOPHISTICATION AND REDUCTION

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Abstract: The mental logic theory proposes that people think following a logic. That logic is not the classical one, but a special mental logic. Mental logic is akin to classical logic. However, there are important differences between them. One of those differences is that, while in classical logic Modus Tollendo Tollens is an acceptable inference, in mental logic only sophisticated individuals usually derive the right conclusion in that inference. This paper, following Carnap's framework, tries to offer a reduction sentence to gradually confirm that thesis from the mental logic theory. The paper takes two exceptions into account: the cases in which unsophisticated individuals also apply Modus Tollendo Tollens, and the cases in which even sophisticated individuals cannot apply that very inference.

Keywords: Carnap; logical sophistication; mental logic; Modus Tollendo Tollens; reduction sentence

INTRODUCTION

Today it is hard to claim that naïve (with no logical training) people make inferences in accordance with classical propositional calculus in a natural way. The logical system that can be derived from works such as those of Gentzen[1] does not seem to be the criterion to describe what individuals do when inferring conclusions. Several results in reasoning tasks showing that are to be found in the

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[1] Gerhard Gentzen, "Untersuchungen über das logische Schließen I", *Mathematische Zeitschrift*, vol. 39, nr. 2, 1934, pp. 176–210; Gerhard Gentzen, "Untersuchungen über das logische Schließen II", *Mathematische Zeitschrift*, vol. 39, nr. 3, 1935, pp. 405–431.

literature.[2] However, in the field of cognitive science, some theories proposed a thesis related to that but different from it: the human mind is linked to a logic, but that logic is not classical propositional logic.[3]

One of those proposals is that of the mental logic theory.[4] Following this theory, the logic in the human mind is similar to that of propositional calculus. Nevertheless, there are important differences between the two logics. Not all of the rules that exist in classical calculus are correct in mental logic. On the other hand, some rules from classical logic that are admitted cannot be always applied in mental logic.[5] Besides, mental logic does not understand contradictions as classical logic.[6] Likewise, the theory provides that the premises from which inferences are made are not only the explicit ones. Individuals also take into account premises offered by general knowledge or pragmatics in daily life reasoning processes.[7]

Nonetheless, another relevant concept in the mental logic theory is that of logical sophistication.[8] According to the proponents of the theory, there are sophisticated people who are able to make valid inferences in propositional calculus that most individuals cannot often make. This is the aspect this paper will deal with, particularly with the case of Modus Tollendo Tollens. Modus Tollendo Tollens is an inference rule correct in classical logic. However, following the mental logic theory, it is not one of the rules people naturally apply. It is not a

[2] E.g., Ruth M. J. Byrne, "Suppressing valid inferences with conditionals", *Cognition*, vol. 31, nr. 1, 1989, pp. 61–83; Philip N. Johnson-Laird, "Against logical form", *Psychologica Belgica*, vol. 50, nr. 3/4, 2010, pp. 193–221; Isabel Orenes, Philip N. Johnson-Laird, "Logic, models, and paradoxical inferences", *Mind & Language*, vol. 27, nr. 4, 2012, pp. 357–377; Peter C. Wason, "Reasoning", in Brian Foss (comp.), *New Horizons in Psychology*, Harmondsworth (Middlesex), Penguin, 1966, pp. 135–151; Peter C. Wason, "Reasoning about a rule", *Quarterly Journal of Experimental Psychology*, vol. 20, nr. 3, 1968, pp. 273–281.

[3] E.g., Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998; Lance J. Rips, *The Psychology of Proof: Deductive Reasoning in Human Thinking*, Cambridge, Massachusetts Institute of Technology (MIT) Press, 1994.

[4] E.g., M.D.S. Braine, D.P. O'Brien (eds.), *Mental Logic*.

[5] E.g., Martin D.S. Braine, David P. O'Brien, "The theory of mental-propositional logic: Description and illustration", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*. Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 79–89.

[6] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic,* Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 206.

[7] See also, e.g., Martin D.S. Braine, David P. O'Brien, "How to investigate mental logic and the syntax of thought", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 46.

[8] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic,* Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 223.

'Core Schema',[9] that is, one of the essential rules of mental logic individuals frequently use. But the theory proposes that there are sophisticated people that can easily apply it.

The aim of the present paper is to try to present a protocol to contrast or verify the idea of logical sophistication in the mental logic theory. To this effect, Carnap's framework[10] will be adopted. As other philosophers of science,[11] Carnap[12] considers total verification of hypotheses to be unachievable. But Carnap thinks that scientists can move forward and progressively confirm their theories by checking whether or not certain conditional relations happen.[13] To show how that process of progressive confirmation should be in the case of logical sophistication is the main goal here.

The first section of the paper will describe the general approach of the mental logic theory, and the manner it understands Modus Tollendo Tollens. Then, it will indicate the provisional conditional sentence that could be built from Carnap's framework to try to confirm the idea of logical sophistication in the mental logic theory. The third section will explain that, because of some experimental results already recorded in the literature, to propose biconditional relations in the process of progressive confirmation of that idea would be difficult. Finally, the paper will show how the provisional conditional sentence should be modified to address some difficulties the hypothesis about logical sophistication face: the particular cases in which sophisticated individuals cannot use the rule of Modus Tollendo Tollens.

MENTAL LOGIC, MODUS TOLLENDO TOLLENS, AND LOGICAL SOPHISTICATION

As said, several components separate mental logic from propositional logic. First, not all the rules in classical logic are correct in mental logic. A well-known, particular case is that of the disjunction introduction rule. That rule is (1).

$$(1) \qquad p \therefore p \vee q$$

Where '$\therefore$' represents logical deduction and '$\vee$' stands for disjunction.

[9] E.g., Martin D.S. Braine, David P. O'Brien, "The theory of mental-propositional logic: Description and illustration", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*. Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 79–89.

[10] E.g., Rudolf Carnap, "Testability and meaning", *Philosophy of Science*, vol. 3, nr. 4, 1936, pp. 419–471; Rudolf Carnap, "Testability and meaning – Continued", *Philosophy of Science*, vol. 4, nr. 1, 1937, pp. 1–40; Rudolf Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*, Chicago, The University of Chicago Press, 1947.

[11] E.g., Mario Bunge, *La investigación científica*, Barcelona, Ariel, 1985, p. 27; Karl Popper, *The Logic of Scientific Discovery*, London, Routledge, 2002, p. 32.

[12] R. Carnap, "Testability and meaning", p. 425.

[13] *Ibidem*, pp. 419–471.

In propositional calculus, if p is true, it is possible to deduce that $p \vee q$ is true, too. However, following the literature, people do not usually accept inferences such as (1).[14] For this reason, the mental logic theory does not admit (1) as a Core Schema.[15]

On the other hand, some rules that the mental logic theory accepts have limitations that they do not have in classical logic. This is the case of the conjunction introduction rule:

(2) $\{p, q\} \therefore p \wedge q$

Where '$\wedge$' denotes conjunction.

Schema (2) has the limitation that it can only be applied if it is necessary to continue with the deductive process. Without the limitation, it could lead to infinite derivations such as those indicated in (3), (4), and (5).[16]

(3) $\{p, q, p \wedge q\} \therefore p \wedge q \wedge q$

(4) $\{p, q, p \wedge q, p \wedge q \wedge q\} \therefore p \wedge q \wedge q \wedge q$

(5) …

Another important point is that contradictions do not allow deducing the same as in propositional calculus. In this last calculus, given a contradiction, by virtue of the principle of explosion (*ex falso quodlibet*, or *ex contradictione quodlibet*), any conclusion follows. (6) shows this.

(6) $p \wedge \neg p \therefore X$

Where '$\neg$' indicates negation and 'X' is any proposition or formula.

But in mental logic any proposition or formula cannot be inferred from a contradiction. The only role of contradictions in mental logic is that they make it explicit that one of the previous assumptions leading to the contradiction is false.[17]

[14] I. Orenes, P.N. Johnson-Laird, "Logic, models, and paradoxical inferences", pp. 357–377.

[15] Martin D.S. Braine, David P. O'Brien, "The theory of mental-propositional logic: Description and illustration", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*. Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 87–88.

[16] *Ibidem*, pp. 80–83.

[17] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D. S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 206.

One more relevant aspect is that, according to the mental logic theory, real inferences in daily life include hidden premises that are not explicitly mentioned. For example, people usually make inferences such as (7).

(7) If they go to Italy, then they will be happy

They go to Rome

Therefore, they will be happy

Strictly speaking, the conclusion in (7) does not get derived from its premises. The logical form of (7) is (8).

(8) $\{p \rightarrow q, r\} \therefore q$

Where '$\rightarrow$' represents the logical conditional.

Inference (8) is not admissible in classical propositional logic, or, in principle, in mental logic. Proposition q cannot be deduced from premises $p \rightarrow q$ and r. However, the mental logic theory can explain this fact by virtue of its idea that real inferential processes also refer to premises general knowledge or pragmatics give.[18] In particular, in the case of (8), the hidden premise can be (9).

(9) If they go to Rome, then they go to Italy

Hence, the actual structure of the inference is not (8), but (10).

(10) $\{p \rightarrow q, r \rightarrow p, r\} \therefore q$

And (10) can be accepted in both classical calculus and mental logic. This is because both of them admit Modus Ponendo Ponens, that is, (13). So, both logical systems enable to infer p from $r \rightarrow p$ and r, and q from $p \rightarrow q$ and p.

Nevertheless, the most important component of the mental logic theory for the present paper is that it includes the concept of logical sophistication as well.[19] There are certain inferences that are hard for individuals. But sophisticated people appear to be able to make them easily. That is the case of Modus Tollendo Tollens, that is, (11).

[18] See also, e.g., Martin D.S. Braine, David P. O'Brien, "How to investigate mental logic and the syntax of thought", in Martin D. S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 46.

[19] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D. S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, p. 223.

(11) $\{p \rightarrow q, \neg q\} \therefore \neg p$

Modus Tollendo Tollens is not an essential rule, that is, a Core Schema, in the mental logic theory.[20] However, sophisticated individuals can apply it. According to the theory, Modus Tollendo Tollens is not impossible, but it is difficult. For this reason, sometimes only sophisticated people use it. (11) is hard because it needs, in a similar way as in propositional logic, the Reductio ad Absurdum strategy.[21] This strategy consists of introducing, in the set of premises in (11), p as an assumption. This leads to a contradiction such as that (12) reveals.

(12) $\{p \rightarrow q, \neg q, p\} \therefore q \wedge \neg q$

The conclusion in (12) is contradiction $q \wedge \neg q$ by virtue of Modus Ponendo Ponens and the conjunction introduction rule (as indicated, those two rules are valid in both classical propositional calculus and mental logic). But this contradiction does not lead in mental logic to any result. It only informs that one of the premises is false. In the case of (12), it is justified to think that the false datum is p, since it is an assumption. In this way, as in classical logic, although for different reasons, the conclusion can be that in (11), that is, $\neg p$.

Nonetheless, a skeptical cognitive scientist might consider the concept of logical sophistication to be just a hypothesis. Thus, it seems to be opportune to present a protocol to empirically contrast the concept. The rest of this paper is devoted to the elaboration of that protocol. The paper assumes Carnap's framework.[22]

A REDUCTION SENTENCE RELATED TO LOGICAL SOPHISTICATION

The mental logic theory rejects the material interpretation of the conditional (i.e., the way classical logic understands it). Following the theory, the basic or essential rules for the conditional are two.[23] One of them is Modus Ponendo Ponens, that is, (13).

[20] For explanations of the difficulties of Modus Tollendo Tollens from other theoretical frameworks, see, e.g., Ruth Byrne, Philip N. Johnson-Laird, "*If* and the problems of conditional reasoning", *Trends in Cognitive Science*, vol. 13, nr. 7, 2009, pp. 282–283.

[21] See also, e.g., David P. O'Brien, "Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013)", *Universum*, vol. 29, nr. 2, 2014, p. 230.

[22] E.g., R. Carnap, "Testability and meaning", pp. 419–471; R. Carnap, "Testability and meaning – Continued", pp. 1–40; R. Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*.

[23] E.g., Martin D.S. Braine, David P. O'Brien, "The theory of mental-propositional logic: Description and illustration", in Martin D. S. Braine, David P. O'Brien (eds.), *Mental Logic*. Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 80–81.

$$(13) \qquad \{p \rightarrow q, p\} \therefore q$$

The other one, which has some restrictions to be applied,[24] is the conditional introduction rule:

$$(14) \qquad q \therefore p \rightarrow q$$

Beyond this, the conditional does not have the other characteristics it has in propositional logic.

However, given that this paper is set in a metatheoretical perspective based on Carnap's approach, the confirmation protocol that will be presented for the concept of sophistication will understand the conditional as Carnap[25] does, that is, as a material conditional (i.e., as in classical logic). It is important to make this point clear, but it does not have an influence on the argumentation below. For this reason, the symbol to express conditional relation will keep being the same in that argumentation.

Carnap's framework has already been used for other cognitive theories. That is the case of the theory of mental models.[26] For the latter theory, protocols akin to the one that will be described here have been proposed.[27] As far as the mental logic theory is concerned, from what has been explained above, it is possible to assume that, if a human being is a sophisticated reasoner, that human being should be able to correctly use Modus Tollendo Tollens. So, if a person does not come to the conclusion expected in Modus Tollendo Tollens, that person cannot be sophisticated. This can be expressed by means of conditional relations if the following equivalences are provided:

$H =_{df}$ to be a human being

$S =_{df}$ to be a sophisticated person

[24] E.g., Martin D. S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 204–211.

[25] R. Carnap, "Testability and meaning", pp. 419–471; R. Carnap, "Testability and meaning – Continued", pp. 1–40; R. Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*.

[26] E.g., Ruth Byrne, Philip N. Johnson-Laird, "*If* and *or*: Real and counterfactual possibilities in their truth and probability", *Journal of Experimental Psychology: Learning, Memory, and Cognition*, vol. 46, nr. 4, 2020, pp. 760–780; Philip N. Johnson-Laird, Ana Cristina Quelhas, Célia Rasga, "The mental model theory of free choice permissions and paradoxical disjunctive inferences", *Journal of Cognitive Psychology*, 2021. https://doi.org/10.1080/20445911.2021.1967963

[27] E.g., Miguel López-Astorga, "The definition of modulation and its reduction sentences", *Open Insight*, vol. XIII, nr. 27, 2022, pp. 106–118.

M $=_{df}$ to infer the correct conclusion in inferences with the structure of Modus Tollendo Tollens

The conditional relations between these three predicates would be those in (15).

(15) $\forall x\ [Hx \rightarrow (\neg Mx \rightarrow \neg Sx)]$

Where '∀' is the universal quantifier.

Formula (15), which corresponds to first order predicate calculus, indicates that if x is a human being, then if x does not come to the right conclusion in inferences with the structure of Modus Tollendo Tollens, then x is not a sophisticated person. (15) is equivalent to (16).

(16) $\forall x\ [Hx \rightarrow (Sx \rightarrow Mx)]$

And, according to Carnap,[28] a sentence with the logical form of (16) is a reduction sentence for M. As mentioned, Carnap does not think that it is possible to come to absolute verifications. However, Carnap's view[29] is different from that of Popper.[30] Following Popper, hypotheses should undergo falsification processes. On the other hand, Carnap[31] speaks about progressive confirmation. In this way, (16) allows incrementally checking whether or not the individuals that are sophisticated correctly apply Modus Tollendo Tollens. The more corroborated (16), the more confirmed its conditional relations.

A relevant characteristic of reduction sentences is that, as Carnap[32] points out, there is a requirement for them. In the case of (16), the requirement would be (17).

(17) $\exists x\ (Hx \wedge Sx)$

Where '∃' expresses existential quantification.

Given the predicates assumed above, requirement (17) provides that there should be at least a human being (Hx) who can be deemed as a sophisticated person (Sx). That is not a problem in this case, since it seems to be possible to find people that, in principle, appear to be sophisticated.

[28] R. Carnap, "Testability and meaning", p. 442.
[29] *Ibidem*, pp. 419–471.
[30] K. Popper, *The Logic of Scientific Discovery*.
[31] R. Carnap, "Testability and meaning", pp. 419–471.
[32] *Ibidem*, p. 442.

BILATERAL REDUCTION SENTENCES ARE NOT POSSIBLE FOR LOGICAL SOPHISTICATION

At the moment, (16) is the provisional reduction sentence that can lead the confirmation process of the hypothesis of the mental logic theory about logical sophistication. Now, possible exceptions have to be reviewed. The exceptions can be of two kinds:

(A) There can be individuals able to apply Modus Tollendo Tollens without being sophisticated

(B) There can be particular inferences of Modus Tollendo Tollens that are not made even by sophisticated people

This section will address (A); the next section will deal with (B).

The situations that (A) indicates are possible. Sometimes, unsophisticated individuals are able to use Modus Tollendo Tollens. This can be for several reasons. For example, the conditionals expressing an obligation seem to facilitate Modus Tollendo Tollens,[33] when the conditional is perfected, that is, understood as a biconditional, Modus Tollendo Tollens is also rightly applied,[34] and general knowledge can lead to the expected result in that very inference.[35]

Most these cases are not a problem within the mental logic theory. The theory can explain them. For instance, in the case of the action of general knowledge, it is possible to resort to the idea of implicit premises. (18) illustrates this.

(18) If they go to Paris, then they go to France
They do not go to France

Therefore, they do not go to Paris

Inference (18) can be made by unsophisticated average individuals. This is because general knowledge provides an additional, hidden premise: (19).

[33] E.g., Ruth Byrne, *The Rational Imagination: How People Create Alternatives to Reality*, Cambridge, The Massachusetts Institute of Technology (MIT) Press, 2005; Marcos Cramer, Steffen Hölldobler, Marco Ragni, "When are humans reasoning with Modus Tollens?" *Proceedings of the Annual Meeting of the Cognitive Science Society*, vol. 43, nr. 43, 2021, pp. 2337–2343.

[34] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D. S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 223.

[35] E.g., Miguel López-Astorga, "Chrysippus' *indemonstrables* and mental logic", *Croatian Journal of Philosophy*, vol. 15, nr. 43, 2015, pp. 9–11.

(19) If they do not go to France, then they do not go to Paris

Accordingly, the real structure of (18) is (20).

(20) If they go to Paris, then they go to France
If they do not go to France, then they do not go to Paris
They do not go to France

Therefore, they do not go to Paris

But (20) does not require the application of Modus Tollendo Tollens, but the basic rule Modus Ponendo Ponens. This last schema should be applied to the second premise ('if they do not go to France, then they do not go to Paris') and the third premise ('they do not go to France'). Thereby, the application of Modus Tollendo Tollens is just apparent.[36]

However, beyond the machinery the mental logic theory has to solve the mentioned situations, it cannot be stated that only sophisticated individuals can give the correct answer in inferences with the structure of Modus Tollendo Tollens. There are cases in which unsophisticated people can also do that. Hence, (21) cannot be admitted.

(21) $\forall x\ [Hx \rightarrow (Mx \rightarrow Sx)]$

It is possible to apply Modus Tollendo Tollens (although that is in an apparent way) without being a sophisticated person.

This is important because if (21) could be accepted, (16) and (21) would allow building (22).

(22) $\forall x\ [Hx \rightarrow (Sx \leftrightarrow Mx)]$

Where '$\leftrightarrow$' stands for biconditional logical relation.

Within Carnap's framework,[37] a sentence with the logical form of (22) would be a bilateral reduction sentence for M. But, because (21) cannot be admitted, (22) is not valid either. This makes it clear that the process of progressive confirmation of the concept of logical sophistication should only consider the conditional relations (16) provides. The biconditional relation (22) presents should be ignored.

[36] For similar accounts, e.g., *ibidem*.
[37] R. Carnap, "Testability and meaning", pp. 442–443.

SOPHISTICATION AND L-FALSITY

What (B) indicates needs to be analyzed, too. The circumstances it describes can also be the case. There are inferences with the structure of Modus Tollendo Tollens that even sophisticated individuals cannot accept. (23) can be an example.

(23) "If she played a musical instrument then she didn't a flute.
She played a flute.
So, she didn't play a musical instrument".[38]

One cannot expect that people admit that the conclusion in (23) follows its premises, even if the logical structure is correct.[39] Nevertheless, in principle, that is not a problem for the mental logic theory. From this last theory, there is an implicit premise, which is (24).

(24) "…if she played a flute, she played a musical instrument…"[40]

The second premise in (23) and (24) enable to come to 'she played a musical instrument', which is the opposite of what the conclusion in (23) affirms. The contradiction reveals that at least one of the premises is unacceptable. Given that the second one in (23) is a fact and that (24) comes from general knowledge, one might think that the false premise is the first one in (23). If this last premise is removed, the conclusion in (23) cannot be inferred.[41]

What is a problem is that provisional reduction sentence (16) cannot lead the protocol of progressive confirmation. This is because there are circumstances in which even sophisticated people cannot use Modus Tollendo Tollens. It is hence necessary to review (16).

In this regard, the particular characteristics of inferences such as (23) should be identified. Without ignoring the accounts in this way that can be given from the mental logic theory,[42] what makes (23) different seems to be its conditional. It includes two clauses that, if they are negated at once, they describe an impossible situation: no one can play a flute without playing a musical instrument.

[38] Philip N. Johnson-Laird, "Against logical form", p. 201.

[39] *Ibidem*, pp. 201–202.

[40] *Ibidem*, p. 202.

[41] Interpretations of the role of hidden premises in theories such as the mental logic theory can be found in different works, whether or not those works assume those theories. Beyond explanations such as that in P. N. Johnson-Laird, "Against logical form", pp. 203–204, see, e.g., Miguel López-Astorga, "An essentially syntactic and formal theory is still possible", *Pragmalingüística*, vol. 25, 2017, 336–337.

[42] E.g., Martin D.S. Braine, David P. O'Brien, "A theory of if: A lexical entry, reasoning program, and pragmatic principles", in Martin D.S. Braine, David P. O'Brien (eds.), *Mental Logic*, Mahwah, Lawrence Erlbaum Associates, Inc., Publishers, 1998, pp. 199–244.

Within Carnap's philosophy, an impossible situation is a L-false situation. Following the semantic method of extension and intension Carnap proposed,[43] it is necessary to consider state-descriptions. In state-descriptions all the atomic propositions are to be found. The differences between state-descriptions are the truth value of their atomic propositions. Thereby, if $S\text{-}D_1$ and $S\text{-}D_2$ are two state descriptions, they are different if and only if in $S\text{-}D_1$ there is at least one atomic proposition that has a different truth value in $S\text{-}D_2$.

Based on this, Carnap[44] develops the concept of L-falsity. A proposition is L-false if and only if the language, in this case, English, makes it false. The language can prevent a proposition from being true by virtue of the meaning and, accordingly, the words composing the particular proposition. Therefore, a proposition including the idea that she plays a flute rejecting the idea that she plays a musical instrument is L-false: language does not allow that situation: So, a proposition is L-false if and only if it is false in all the state-descriptions, or 'there is not at least one state-description in which it is true.'

The concept of state-description has already been used in cognitive science in order to explain what occurs in some reasoning problems.[45] Here it can help qualify (16). In this reduction sentence, predicate M needs to be replaced with the following predicate:

$M^{(\neg\text{L-false})} =_{df}$ to infer the correct conclusion in inferences with the structure of Modus Tollendo Tollens whose conditional does not lead to a L-false situation if its two clauses are negated

That transforms (16) into (25).

$$(25) \qquad \forall x\, [Hx \rightarrow (Sx \rightarrow M^{(\neg\text{L-false})}x)]$$

Formula (25) would be the new reduction sentence, which, in this case, would be a reduction sentence for $M^{(\neg\text{L-false})}$. It clarifies that the inferences of Modus Tollendo Tollens sophisticated individuals make are those in which the negation of the two clauses of the conditional does not imply a L-false situation.[46]

CONCLUSIONS

According to the mental logic theory, average individuals do not often derive the right conclusion in inferences with the structure of Modus Tollendo Tollens.

[43] R. Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic*.

[44] *Ibidem*, p. 11.

[45] E.g., Miguel López-Astorga, "Wason Selection Task and a semantics based on state-descriptions", *Problemos*, vol. 101, 2022, pp. 8–17.

[46] From the theory of mental models, the idea of an impossible situation when the two clauses of a conditional are negated is also present. However, the latter theory does not speak about state-descriptions or logical formulae, but mental models. For the particular case of (23), see, e.g., Philip N. Johnson-Laird, "Against logical form", pp. 201–204, where theories such as the mental logic theory are challenged.

That is a hard task requiring a certain degree of sophistication in individuals. They should be able to apply the Reductio ad Absurdum strategy. This enables to build a provisional reduction sentence providing that if a person is a human being, then if that person is sophisticated, then that very person has to be able to correctly apply Modus Tollendo Tollens.

However, the converse conditional relation between the two last predicated (to be sophisticated and to be able to correctly apply Modus Tollendo Tollens) cannot be accepted. It is not the case that if a person is a human being, then if that person is able to correctly apply Modus Tollendo Tollens, then that very person is sophisticated. There are particular situations in which unsophisticated individuals can come to the right conclusion in an inference of Modus Tollendo Tollens. This prevents from offering a bilateral reduction sentence related to those concepts. It cannot be claimed that if a person is a human being, then that person is sophisticated if and only if that very person correctly applies Modus Tollendo Tollens. This implies that, in the gradual process of confirmation of the hypothesis of logical sophistication in the mental logic theory, in principle, it is necessary to keep using only the previous reduction sentence.

But there is another problem: There are inferences of Modus Tollendo Tollens that even sophisticated people do not make. For this reason, actually the provisional reduction sentence has to be modified.

The inferences with the structure of Modus Tollendo Tollens that are not made even by sophisticated individuals appear to share a feature. That feature reveals the way to modify the provisional reduction sentence. Those last inferences of Modus Tollendo Tollens usually include conditionals with clauses that, if both of them are negated at the same time, describe an impossible world. This fact allows resorting to other Carnap’s proposals, too. In particular, it enables to consider concepts such as that of L-falsity and a semantics of state-descriptions. Thus, the provisional reduction sentence can be qualified by indicating that the predicate for which it is a reduction sentence refers to inferences with the structure of Modus Tollendo Tollens having a characteristic: if the two clauses of their conditionals are negated, that does not lead to an impossible situation.

Thereby, a new provisional reduction sentence to progressively confirm that sophisticated people tend to properly apply difficult inferences such as Modus Tollendo Tollens can be obtained. Nevertheless, if the analysis of everything the concept of sophistication in the mental logic theory implies continues, perhaps more modifications necessary to the reduction sentence are discovered.

Acknowledgments

PIA Ciencias Cognitivas, Centro de Investigación en Ciencias Cognitivas, Instituto de Estudios Humanísticos, Universidad de Talca.

Fondo Fondecyt de Continuidad para Investigadores Senior, código FCSEN2102, Universidad de Talca.

Fondequip (Programa de Equipamiento Científico y Tecnológico) 2019, código EQM190153.