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IS THE ARROW'S FLIGHT A PROCESS?

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Abstract: Zeno's famous arrow's paradox has troubled philosophers for a long time. In the aftermath of Russell's discussion of the paradox in terms of the calculus, I argue that the paradox leaves a lingering question as to how our everyday, pre-theoretical notions of the motion of objects (such as arrows) intermesh with the mathematical physics thought to fully account for them. Starting from Russell and Salmon's reformulations of the arrow paradox in terms of 'at-at' theories of motion, I argue that such solutions can only account for our pre-theoretical intuitions if supplemented ontologically, by something in the vein of (though perhaps not necessarily identical with) Whitehedian processes. I then discuss the suitability of this approach to the arrow paradox, and end by exploring ontological and metaontological concerns one might raise about whether this is a viable way out of paradox.

Keywords: the arrow paradox, ontology of processes, Zeno of Elea, motion, continuity.

1. INTRODUCTION: ZENO'S ARROW PARADOX MEETS ANALYSIS

Perhaps Zeno's arrow paradox is still with us. Minimally, I aim to suggest that current approaches have not yet fully disposed of the paradox and, in particular, that Salmon's approach of the paradox (as first reformulated by Russell) calls for supplementation and revision. I further argue that such supplementation and revision may plausibly be carried out along the lines suggested by Whitehead in *Process and Reality*. And that the resulting view is competitive as an account that aims to solve Zeno's (amended) arrow's paradox.

I end, however, with a methodological remark concerning whether a metaphysical solution, that posits processes as extended and fundamental, is the right kind of answer to a conceptual paradox about continuity, motion, and object endurance across spacetime regions, or whether an ontology-ideology tradeoff, while useful in some research contexts, may not always be fruitful or illuminating.

Consider, to begin with, Aristotle's rendering of Zeno's arrow paradox[1]:

> 'The third is … that the flying arrow is at rest, which result follows from the assumption that time is composed of moments … he says that if everything when it occupies an equal space is at rest, and if that which is in locomotion is always in a now, the flying arrow is therefore motionless.' (Aristotle *Physics*, 239b30)

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[1] Quoted after Huggett (2019), section 3.3.

Aristotle puts it in terms that presuppose a host of metaphysical questions: that time exists (and how it can do so), that it is composed, that instants compose it, that there is some connection between the time it takes an arrow to move from one region of space to another (and perhaps we could model that lapse as a region as well), that there is some distinct present instant, such that we could identify other instants as past and other as future with respect to it, and so on. Perhaps a clearer, if briefer, presentation of the arrow paradox appears in Diogenes Laertius' *Lives of Famous Philosophers*, Book IX, Ch. 72: 'What is in motion moves neither in the place it is nor in one in which it is not'.[2] Unlike the previous one, this formulation does not presuppose that time is composed of instants and seems much closer to everyday intuitive representations.

How should we best characterize the paradox? Consider that before reaching its target headed from its point of origin, the arrow has to first reach half that distance, and to do that the arrow first has to reach a quarter of the whole distance, one eighth, and so on. If one thinks that the trouble consists merely in Zeno's not having grasped the notion of a convergent series (such as $1/2^n$), that initial reaction (which I will return to) seems hasty. It is true that this is partly the source of Zeno's puzzle, but even after having learned calculus it seems as though a question still lingers.[3] Thus, in one of his appraisals, Russell (1917, pp. 81-82) writes[4]:

> 'Zeno was concerned, as a matter of fact, with three problems, each presented by motion, but each more abstract than motion, and capable of a purely arithmetical treatment. These are the problems of the infinitesimal, the infinite, and continuity. To state clearly the difficulties involved, was to accomplish perhaps the hardest part of the philosopher's task. This was done by Zeno. From him to our own day, the finest intellects of each generation in turn attacked the problems, but achieved, broadly speaking, nothing. In our own time, however, three men – Weierstrass, Dedekind, and Cantor – have not merely advanced the three problems, but have completely solved them.'

If Russell were correct, nothing would now be left of the paradox but a historically important moment for raising questions that Weierstrass, Dedekind and Cantor solved. As Russell puts it elsewhere,

> 'The solution lies in the theory of continuous series: we find it hard to avoid supposing that, when the arrow is in flight, there is a next position occupied at the next moment; but in fact there is no next position and no next moment, and when once this is imaginatively realised, the difficulty is seen to disappear.' (Rusell 1970, p.51).

Is there any lingering concern? I will argue there is.

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[2] *Ibidem*.

[3] In fact, this is only a partial first approximation. As Grünbaum (1970, p. 187) argues, 'the theory of infinite divisibility has been used fallaciously in an attempt to deduce Zeno's metrical paradox.'

[4] This is also the attitude towards Zeno's arrow paradox suggested by Huggett (2019).

2. RUSSELL ON ZENO

On that same page as quoted previously, Russell addresses the paradox quite differently:

> 'Thus, suppose we consider a period consisting of a thousand instants, and suppose the arrow is in flight throughout this period. At each of the thousand instants, the arrow is where it is, though at the next instant it is somewhere else. It is never moving, but in some miraculous way the change of position has to occur *between* the instants, that is to say, not at any time whatever.'

This is notably different because it does not concern the continuity of space alone, but of space as is traversed by an arrow in flight. Physical motion is important here because instantaneous velocity is conceived of as a continuous magnitude, and the relationship between instants (or points in space, or regions in time, or regions of space) does not account for physical motion fully (unless one were a Pythagorean of sorts, which most of us depart from being, perhaps unlike Zeno). In other words, the quandary of what happens to the arrow's motion isn't fully resolved by bringing up points about the continuity of mathematical space (however construed). How the mathematics of spacetime applies to the arrow's motion (and its velocity) is key here.[5] As Salmon construes Russell's remarks:

> 'instantaneous velocity is defined as the limit, as we take decreasing time intervals, of the non-instantaneous average velocity with which the object traverses what is – in the case of nonzero values – a nonzero stretch of space. Thus in the definition of instantaneous velocity, we employ the concept of non-instantaneous velocity, which is precisely the problematic concept from which the paradox arises. To put the same point in a different way, the concept of instantaneous velocity does not genuinely characterize the motion of an object at an isolated instant all by itself, for the very definition of instantaneous velocity makes reference to neighboring instants of time and neighboring points of space. To find an adequate resolution of the flying arrow paradox, we must go deeper.' (Salmon 1984, p.152)

At this point, it might be possible to retort, and this seems to be the way out that Russell prefers, that the mathematics of characterizing instantaneous velocity at the limits should be considered separately from the pre-theoretical representations we might have of motion, arrows and the like. Whatever paradox besets our pre-theoretical notions, it does not carry over to the mathematical treatment of motion in classical analysis.

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[5] To illustrate a similar take, here is Whitehead's remark in discussing James on Eleatic paradoxes: 'James also refers to Zeno. In substance I agree with his argument from Zeno; though I do not think that he allows sufficiently for those elements in Zeno's paradoxes which are the product of inadequate mathematical knowledge. But I agree that a valid argument remains after the removal of the invalid parts.' (Whitehead 1978, p.68).

The official view, left outstanding, would have it that there is no question concerning what happens with the arrow in flight between instants (nor between adjacent regions of space). There is no fact of the matter beyond specifying the value of instantaneous velocity at a given instant in time and at a specific region of space. This 'at-at' theory is aptly described by Salmon:

> 'According to the "at-at" theory, to move from A to B is simply to occupy the intervening points at the intervening instants. It consists in being at particular points of space at corresponding moments. There is no additional question as to how the arrow gets from point A to point B; the answer has already been given – by being at the intervening points at the intervening moments. The answer is emphatically not that it gets from A to B by zipping through the intermediate points at high speed. Moreover, there is no additional question about how the arrow gets from one intervening point to another – the answer is the same, namely, by being at the points between them at the corresponding moments. And clearly, there can be no question about how the arrow gets from one point to the next, for in a continuum there is no next point.' (Salmon 1984, p. 153)

Notice Salmon's emphasis: there is no question, we should refrain from importing our everyday representations here. This is because, if we do, the paradox reemerges. This provides a specific profile to the reformulation of Zeno's arrow paradox through the lens of the 'at-at' theory: it concerns how to reconcile our everyday, partially pre-theoretical, notions of space and time with mechanics and the continuity of space and time it (seemingly) relies on. In fact, to put the point in terms of continuity would be to narrow the issue excessively: if we were concerned with discrete magnitudes rather than continuous ones, the answer would have to be the same according to this view: no in-between's, only the correlation between given instants in time and given values for instantaneous velocity (and, correspondingly, for regions of space).

Salmon then uses Russell's view to his own ends: 'this solution can – if I am right – be extended in a direct fashion to provide a resolution of the problem of [causal] mark transmission' (Salmon 1984, p.153). On the 'at-at' view, 'ability to transmit a mark can be viewed as a particularly important species of constant conjunction – the sort of thing Hume recognized as observable and admissible' (*ibidem*, p.146). Salmon's view, however, is in stark contrast to Russell's own animadversions towards causation:

> 'a great difficulty is caused by the temporal contiguity of cause and effect which the definition asserts. No two instants are contiguous, since the time-series is compact; hence either the cause or the effect or both must, if the definition is correct, endure for a finite time; indeed, by the wording of the definition it is plain that both are assumed to endure for a finite time. But then we are faced with a dilemma: if the cause is a process involving change within itself, we shall require (if causality is universal) causal relations between its earlier and later parts; moreover, it would seem that only the later parts can be

> relevant to the effect, since the earlier parts are not contiguous to the effect, and therefore (by the definition) cannot influence the effect. Thus we shall be led to diminish the duration of the cause without limit, and however much we may diminish it, there will still remain an earlier part which might be altered without altering the effect, so that the true cause, as defined, will not have been reached, for it will be observed that the definition excludes plurality of causes. If, on the other hand, the cause is purely static, involving no change within itself, then, in the first place, no such cause is to be found in nature, and in the second place, it seems strange – too strange to be accepted, in spite of bare logical possibility – that the cause, after existing placidly for some time, should suddenly explode into the effect, when it might just as well have done so at any earlier time, or have gone on unchanged without producing its effect. This dilemma, therefore, is fatal to the view that cause and effect can be contiguous in time' (Russell 1917, p.184).

And Russell concludes abruptly that even though 'many fairly dependable regularities of sequence occur in daily life '… [t]he principle "same cause, same effect," which philosophers imagine to be vital to science, is therefore utterly otiose.' (*idem*, p.189)

This is why it is important to separate the merits of Russell's approach to Zeno's arrow paradox from his stance with respect to causation. The ensuing difficulty resides in whether it might be best to prune discussion of causal effects altogether when it comes to the arrow's flight. This would bar the intuitively appealing, but ultimately paradoxical, suggestion that the arrow's instantaneous velocity at a given region of space and instant in time causally determines its subsequent motion to a different region of space and instant in time. But, if we find this way of phrasing the arrow's flight paradoxical and feel compelled to avoid it, what is left of our pre-theoretical ways of describing the arrow's flight at all? Have we, by wanting to avoid paradox, prevented ourselves from being able to describe the arrow's flight at all? The next section explores a few options.

3. OPTIONS FACED WHEN DEALING WITH THE PARADOX

Taking stock, three options seem open. One is to embrace paradox, which I set aside. Embracing paradox seems counterintuitive for any who think there are no true contradictions. This is not to say the option is forlorn. Should alternatives prove unviable, we might eventually come to embrace this and reconcile ourselves with its unintuitiveness. However, *pro tanto*, it seems preferable to look for alternatives if any are in the offing.

Another possible view, favored by positivists, is to argue that the question doesn't even make sense. If the debate about whether the (revised) Zeno's arrow paradox concerns issues metaphysical that do not touch upon either purely mathematical notions nor upon causal modeling, why should such issues be of concern to the philosopher of science? Or so might the positivist ask.

If there is a choice between competing linguistic frameworks, such that on one of them it makes no sense to speak about the arrow paradox, and a different linguistic framework on which it does make sense to speak of the arrow paradox, then the choice between them should better be couched in terms of practical convenience and expediency in promoting and sustaining research.[6]

Positivism has its notorious problems which I will not recap here. Let me just say that, specifically when it comes to Russell's reformulation of Zeno's arrow paradox, it *prima facie* seems to fail to do justice to the intuition that questions linger concerning how our pre-theoretical notions of continuity and motion relate to their formal counterparts. (Indeed, a positivist might be hard pressed to even advance that distinction.) If alternatives to positivism are viable, they should first have their day in court.

The third option I see is to identify the arrow's flight with an extended process, irreducible to the events of pointwise motion that compose it. I by no means suggest that these are all the options available, only that in this context they seem to be the prominent ones. This option, I will argue in the next section, is congenial to Whitehead's notion of process.

4. WHITEHEAD ON ZENO

For the purpose of the ensuing discussion, Whitehead matters when it comes to Zeno's arrow paradox for main two reasons. The Whitehedian idea of process seems key to the difference between our pre-theoretical notions of motion, continuity, and causal connection, on the one hand, and the formal 'at-at' view that Russell advances in reply to the arrow. This reply includes two issues: (A) the appeal to processes as ontologically fundamental (at least when it comes to dealing with Zeno's arrow paradox), and (B) the difference between metaphysics and what is afforded as foundations for natural-science approaches to continuity and motion. Whitehead writes:

> 'Eliminating the irrelevant details of the race and of motion–details which have endeared the paradox to the literature of all ages–consider the first half-second as one act of becoming, the next quarter-second as another such act, the next eighth-second as yet another, and so on indefinitely. Zeno then illegitimately assumes this infinite series of acts of becoming can never be exhausted. But there is no need to assume that an infinite series of acts of becoming, with a first act, and each act with an immediate successor, is inexhaustible in the process of becoming. Simple arithmetic assures us that the series just indicated will be exhausted in the period of one second. The way is then open for the

[6] Notice this doesn't rule out embracing paradox, at least not on the face of it. If embracing paradox might be an attitude that proves more fruitful in advancing research (as evaluated *ex post facto*, when the research in question is achieved and its fruits can be reaped), then a positivist attitude (though perhaps not a logically positivist one), immune to abhorring contradictions, might be compatible with embracing paradox.

> intervention of a new act of becoming which lies beyond the whole series. Thus this paradox of Zeno is based upon a mathematical fallacy. The modification of the 'Arrow' paradox, stated above, brings out the principle that every act of becoming must have an immediate successor, if we admit that something becomes. For otherwise we cannot point out what creature becomes as we enter upon the second in question. But we cannot, in the absence of some additional premise, infer that every act of becoming must have had an immediate predecessor. The conclusion is that in every act of becoming there is the becoming of something with temporal extension; but that the act itself is not extensive, in the sense that it is divisible into earlier and later acts of becoming which correspond to the extensive divisibility of what has become. In this section, the doctrine is enunciated that the creature is extensive, but that its act of becoming is not extensive.' (1978, p. 69)

This passage includes a number of intriguing remarks, not least of which is the doctrine stated at the end: that 'the creature is extensive, but that its act of becoming is not extensive'. Whitehead's distinction between act and creature, as well as subsuming the paradox of the arrow under the paradox of becoming, form part of a package designed to use (A) and (B) above to address paradoxes.

It has since been disputed that Whitehead's solution might be correct. For Grünbaum, 'Zeno's mathematical· paradoxes are avoided in the formal part of a geometry or chronometry built on Cantorean foundations' (in Salmon 1970, p. 194). And becoming is questionable on its own terms (Grünbaum 1950). For my purposes here, I do not intend to defend Whitehead or to ascertain whether these objections are in the right. Rather, it seems to me that Whitehead's appeal to (A) and (B) is central to a resolution of harmonizing our pre-theoretical representations of space, time, motion (and objects that move) with the notions of continuous physical magnitude and instantaneous velocity typically invoked to address them. It is these notions that matter as Zeno's arrow paradox is construed here.

5. TENTATIVE CONCLUSION: ONTOLOGY AND META-ONTOLOGY

It is important to notice that Whitehead's initial discussion of Zeno's arrow paradox occurs without any essential reference to causal connections (they are discussed extensively later in the same chapter). There is no need to suppose, then, that processes (and acts of becoming) are necessarily causal, for the purpose of dealing with the arrow paradox. The doctrine that acts of becoming are not extensive, however, presupposes the fundamentality of something we would now better term as 'processes', and this fundamentality raises at least two questions, metaphysical and methodological.

First comes the question of whether there is an ontological cost to seeing processes as metaphysically fundamental and grounding (created) events. This is one way of reading Whitehead's remark that the 'true difficulty is to understand how the arrow survives the lapse of time' (1978, pp. 68-69), viz. how to best characterize how we approach the arrow paradox from an ontological standpoint. One, then, faces a dilemma: either take on the cost and solve the puzzle at the price of ontological inflationism, or be left with a live puzzle that cannot be handled in a sparse ontology that does other work notwithstanding. One may choose to opt for the ontological gain so as to smoothen conceptual difficulties. Or one may go on searching for options.

Despite this ground-level debate in ontology, we might perhaps raise a distinct and preliminary methodological point, best relegated to meta-metaphysics. Whitehead's treatment seems to face this quandary: If we do think that one hand (inflated ontology) washes another (conceptual paradox), how can this be so? Can ontological changes indeed solve conceptual difficulties? And, if we grant they may sometime do so (e.g., as with Quine's examples of ontological decision), it might still be in the purview of a context-sensitive methodology to refrain from thinking that inflating ontology disposes of ideological worries in all relevant cases, such as Zeno's arrow paradox.

Oftentimes it is heard that Zeno's paradoxes, especially the arrow paradox, are disposed of by a proper understanding of the mathematical treatment of motion, continuity and/or the succession of instants in time. On the contrary, if the foregoing are on the right track, we are left with a genuine paradox on our hands. The paradox doesn't necessarily concern arrows, motion or continuity. Rather, it concerns how to mesh our pre-theoretical notions of motion, objects, the passing of time and moving from one region of space into another with their mathematical and physical counterparts. Neither ontological maneuvering, nor a focus on the foundations of mathematical physics, fully dispel our quandaries. But this is precisely the level at which Zeno had formulated his arrow paradox: as a rejoinder to our commonsense ways of thinking about how objects move. It would seem, from that standpoint, that Zeno's arrow paradox, while it has been sharpened, hasn't been fully and completely addressed in how our everyday conceptual repertoires interact with scientific pursuits.

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