# ALFRED NORTH WHITEHEAD'S RELATIONAL SPECIAL RELATIVITY

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**Abstract:** The greatest relevance of Alfred North Whitehead's work, in my opinion, has to be related to the interpretation and understanding of the theory of relativity. Whitehead's same philosophy as 'meta-physics,' or 'pan-physics' has to be considered a transition from a logico-mathematical philosophy to a physical philosophy, to a cosmological philosophy rooted in his interpretation of relativity. He understood motion as a structural series of temporal events, avoiding permanent material bodies and refuting any interpretation of four-dimensional space-time as a kind of eternal permanence.

Keywords: special relativity, motion, events, space, time.

# 1. THE RELATIONALITY OF MOTION AND THE RELATEDNESS OF NATURE

The principle of relativity, according to Whitehead, is an ontological principle, not only an epistemological one<sup>1</sup>: the impossibility of knowing the subject of motion is the consequence of the *universal relatedness* of *Nature*, an ontological principle of inter-relationship of every material body with all other material bodies, which so holistically constitute Nature. This constitutive interrelation of all material things explains why our knowledge has limitations in defining individual properties of bodies. Relations between bodies are not 'ideal' relations (as in Leibniz) introduced by the human intellect to order them, but they are real: the fields of forces exist even in the absence of material bodies.

Whitehead gives a new interpretation of the 'principle of (special) relativity of motion,' which tells us that, in the absence of a certainly fixed reference, at rest, it is impossible to know observationally or theoretically which body, between two bodies in reciprocal motion, is at rest and which one is not at rest (in rectilinear and uniform motion), or what is the 'subject of the motion.'

If everything was at rest, everything would appear at rest; if a body for one reference system appears at rest and for another reference appears in motion, then either the body or the reference must be in motion: the relativity of motion implies that at least one *motion exists*. If the Earth is considered at rest, the Sun is moving:

<sup>&</sup>lt;sup>1</sup> A. N. Whitehead, *The Principle of Relativity With Applications to Physical Science*, Cambridge University Press, Cambridge 1922.

motion is projected on another body. In every reference frame system, there is something in motion, something changes. Even if for a body we cannot know whether it is in motion or at rest, we know that there is motion in Nature: motion as a relation is absolute, is invariant. Rest is only a relative rest, that is the situation in which two bodies have the same motion.

A transformation of reference frame transforms a motion of a body in a motion of another body (we have not the same events), but it conserves a certain temporal succession structure of events which we call motion.

The relativity of motion would not occur if there were only static reference frame systems (at rest): it is the consequence of the possibility/need to consider reference frame systems in motion.

Things do not change if we consider non-inertial motions and non-inertial reference frame systems, for which we can state a principle of general relativity of motion.

The principle of general relativity of motion tells us that it is impossible to know observationally or theoretically which body, between two bodies in reciprocal motion, is at rest and which is in motion (even accelerated in any way), or what is the 'subject of motion': then either the body or the reference frame system must be in motion: the general relativity of motion implies that at least one *motion* accelerated in any way *exists*. General arbitrary transformations of reference frame systems can alter the rest or the kind of motion of a body, but they transform an arbitrary motion of a body in the same kind of motion of another body: in different reference frame systems we have not the same events (concerning the same bodies), but a temporal succession structure of events (concerning different bodies), which identified motion is preserved in the transformations.

In Newtonian modern physics, a body or a reference frame system is in an accelerated motion only if a force field acts on it, a field that accelerates it: experimentally, on every body or material reference frame system acts a field of gravitational forces, because gravitation is universal. Strictly speaking all bodies – unless the gravitational field is artificially canceled – and so all reference frame systems move in accelerated motion because they interact with all other bodies in the universe through gravitational force fields. An accelerated reference frame system, that can relativistically modify motion making it a relative thing, can exist only because there are interactions that realize a universal relatedness of Nature. There is no body or reference frame in absolute rest and a general relativity of motion is given.

The principle of general relativity of motion is the consequence of an unavoidable 'solidarity of the universe,' realized through a 'universal relatedness of Nature,' i.e., a 'universal relationality of Nature,' a universal field of (cor-)relations. There are no isolated and separable bodies: Nature is a totality of non-separable parts.

We can have general arbitrary transformations of reference frame systems which can alter the rest or the kind of motion of a body, and which transform an

arbitrary motion of a body in the same kind of motion of another body and preserve a temporal succession structure of events, concerning different bodies, only because of the universal relatedness of Nature. A change in a part of Nature must imply a change in another part.<sup>2</sup>

The principle of relativity is a principle that establishes our ignorance, an epistemological principle that concerns first of all a limit of our knowledge: in general, we cannot attribute to a single body motion as its individual property, but we can only establish it as a relationship between two bodies. We can know only in some special cases, concerning us as moving bodies, which body is moving, but motion is always a relation of a body to other bodies: for a unique existing body, we could not distinguish motion and rest. Motion is a property of Nature as a whole.

This fact, that motion for us is attributable to a body not as an individual property but only as a relationship with another body or with other bodies or relative to a certain chosen point of view (to a 'frame system of reference'), leads Whitehead to conclude that in general we cannot abstract a material body from the existence of the other material bodies with which it is by nature related, i.e., that the universe is not made up of separable material bodies, but rather by bodies that cannot be separated from each other. *Being in relation to other bodies constitutes the essence of a body* and therefore one must consider the universe as an interrelated whole.

## 2. NO MATERIAL BODIES BUT EVENTS. NATURE AS A TEMPORAL PROCESS

However, there's more. The very concept of an individual material body separable from others loses its consistency and can no longer be the basis on which we can constitute the idea of Nature.

If we can affirm that there is a certain relationship of motion between two bodies that can never be completely eliminated, because, even when, from a certain point of view, from a certain frame system of reference, a body is at rest, we must attribute motion to another body – that is, either it is in motion one or the other – what is truly real (invariant for all the reference frame systems) is not the individual body with its supposed properties of motion or stillness that we cannot ascertain, but rather *motion as a relational (collective) property of Nature*. We cannot conceive a body without definite properties, it would be an abstraction. Motion is only a series of events: it is not something identifying or not a body.

Nature then is not made up of stable separable individual material bodies, but Nature is motion as a relation of the parts as events: change, process.

We must include individual material bodies only as *relative parts* of a process, of a change-motion, which, as such, can never be described only in spatial terms, but always implies also a *space-time dimension: a temporal series of events*.

<sup>&</sup>lt;sup>2</sup> A. N. Whitehead, *The Concept of Nature*, Cambridge University Press, Cambridge 1920.

It then explains why in the theory of relativity we must move to a physical description *in a four-dimensional space-time*: because the *Nature* to be described is not made of individual separable, stable in some spatial position, material bodies, but rather *is made of motions, changes, processes, events.* 

There are no more things-in-themselves-substances but only (fields of) events.

Different relations of motion between different parts of Nature imply different temporal relations. Nature is a set of different processes-motions, *a set of different temporalities*.

We understand that *space must also be rethought in terms of time* and we can also understand it in our experience if we do not make abstractions. The weft of space is woven by the vertical warp of the times.

A point in timeless space is not a fundamental entity, but it *is the historical-temporal set of events*, of the processes that happened there: P = [e1, e2, e3, e4, ...]

In 1903, only few years before Whitehead's solution, his scholar *Bertrand Russell* wrote, in the book entitled *The Principles of Mathematics* that a relational theory of space and time should describe the principles of *geometry in terms of sensible entities*.<sup>3</sup> Russell noted that indeed right lines and planes are not such entities, whereas, on the contrary, *metrical (distance) relations are*. Russell went on saying that indeed there is a very complicated method, invented by Leibniz and revised by Frischouf and Peano, by which only distance is fundamental, and the right line is defined from it, even if some of its properties can be introduced only by suitable axioms.<sup>4</sup>

The field of a given distance is the whole space, at variance with the field of the relation that gives rise to a right line which is only such right line itself. Such a relation generating the right line, hence, at variance with the former, makes an intrinsic distinction among space points, that is a distinction that a relational theory has to avoid.

Pieri and others Peano's scholars have tried to formulate geometry starting from the fundamental concept of abstract motion, but they never created an entirely relational theory of geometry.<sup>5</sup>

This kind of approach to a relational theory of geometry did not start from actual physics and involved a change in the fundamental concepts of geometry, metrical geometry concepts replacing descriptive and projective geometry ones at the foundation level.<sup>6</sup>

Whitehead's approach actually overcome this latter abstract (mathematical) one. However, after these works and Whitehead's answer, the relational question

<sup>&</sup>lt;sup>3</sup> B. Russell, *The Principles of Mathematics*, Cambridge University Press, Cambridge 1903 and references therein.

<sup>&</sup>lt;sup>4</sup> L. Couturat, *La Logique de Leibniz*, Paris 1901, p. 420; Frischauf, *Absolute Geometrie nach Johann Bolyai*, Anhang, quoted by Russell; G. Peano, *La geometria basata sulle idee di punto e distanza*, Atti della Reale Accademia delle Scienze di Torino, v. XXXVIII (1902-03), pp. 6-10.

<sup>&</sup>lt;sup>5</sup> See, for example, M. Pieri, "Della geometria elementare come sistema ipotetico-deduttivo," in *Memorie della Reale Accademia delle Scienze di Torino*, v. XLIX (1899), p. 176.

<sup>&</sup>lt;sup>6</sup> B. Russell, An Essaay on the Foundations of Geometry, Cambridge University Press, Cambridge 1897.

was almost completely hidden by the debate on general relativity, and specifically on the problem whether general relativity is actually a relational theory of space, time, and motion. And it was also believed that this latter problem could be reduced to the technical problem of the embedding of the so-called *Mach's principle* within the framework of *general relativity*.<sup>7</sup>

Indeed, even if one accepts the historical analysis given by Gereon Wolters that Mach did not really reject relativity<sup>8</sup>, and even if one accepts the pseudo-Machian formulation of general relativity given by Sciama and others<sup>9</sup>, a relational theory of space, time and motion is a more complex task than this reformulation of general relativity, a task which was realized for special relativity by Whitehead.

It is well known that general relativity has *turned upside down the hierarchy between kinematics (in some interpretation, dynamics) and geometry*: the *kind* of *geometry* which enters in the construction of a physical theory is *no longer given a* 

<sup>&</sup>lt;sup>7</sup> A. Grünbaum, "The Philosophical Retention of Absolute Space in Einstein's General Relativity," in The Philosophical Review, v. LXVI (1957), pp. 525-534; D. W. Sciama, The Unity of the Universe, Faber and Faber, London 1959; D. W. Sciama, The Physical Foundations of General Relativity, Heinemann, London 1969: Cosmology now, ed. by J. Laurie, BBC Publications, London 1973; J. A. Wheeler, "Mach's principle as boundary condition for Einstein's equations," in Gravitation and Relativity, ed. by H. Y. Chiu & W. F. Hoffman, Benjamin, New York 1964, p. 303-349; J. A. Wheeler, Geometrodynamic Steering Principle Reveals the Determiners of Inertia, Princeton preprint 1988; E. Mach, Die Mechanik in ihrer Entwickelung historisch-kritisch dargestellt, Brockhaus, Leipzig 1883; see, for example: G. Boniolo, Mach e Einstein, Armando, Roma 1988; J. B. Barbour, "Relational Concepts of Space and Time," in The British Journal for the Philosophy of Science, v. 33 (1982). pp. 251-274; J. B. Barbour, "Forceless Machian Dynamics," il Nuovo Cimento, v. 26 B (1975), pp. 16-22; J. B. Barbour and B. Bertotti, "Gravity and Inertia in a Machian Framework," il Nuovo Cimento, v. 38 B (1977), pp. 1-27; J. B. Barbour and B. Bertotti, "Mach's Principle and the Structure of Dynamical Theories," Proceedings of the Royal Society of London, v. A 382 (1982), pp. 295-306; D. J. Raine, "Mach's principle and space-time structure," Report on Progress in Physics, v. 44 (1981), pp. 1151-1195 and references therein; F. Hoyle & J. Narlikar, Action at a Distance in Physics and Cosmology, Freeman, San Francisco 1974. For the relations between Mach and Leibniz, see: E. Giannetto, Relativity Theories and Leibniz'Dynamics, mimeographed paper, University of Pavia, Pavia 1992.

<sup>&</sup>lt;sup>8</sup> G. Wolters, Mach I, Mach II, Einstein und die Relativitätstheorie. Eine Fälschung und ihre Folgen, Berlin-New York 1987.

<sup>&</sup>lt;sup>9</sup> D. W. Sciama, "On the origin of inertia," *Monthly Notices of the Royal Astronomical Society*, v. 113 (1953), p. 34; D. W. Sciama, "The Physical Structure of General Relativity," *Reviews of Modern Physics*, v. 36 (1964), pp. 463–469; D. W. Sciama, P. C. Waylen & R. C. Gilman, "Generally Covariant Integral Formulation of Einstein's Field Equations," *Physical Review*, v. 187 (1969), p. 1762–1766; R. C. Gilman, *Physical Review* D, v. 2 (1970), p. 1400; D. Lynden-Bell, "On the Origins of Space-Time and Inertia," *Monthly Notices of the Royal Astronomical Society*, v. 135 (1967), pp. 413–428; H. Goenner, "Mach's Principle and Einstein's Theory of Gravitation," in *Boston Studies in the Philosophy of Science*, v. 6, Reidel, Dordrecht 1970; M. Reinhardt, "Mach's Principle. A critical Review," *Zeitschrift für Naturforschung*, v. 28 A (1972), 529–537; B. L. Altshuler, "Mach's Principle. Part 1. Initial State of the Universe," in *International Journal of Theoretical Physics*, v. 24 (1985), pp. 99–118; D. J. Raine, "Mach's Principle in General Relativity," *Monthly Notices of the Royal Astronomical Society*, v. 171 (1975), pp. 507-528; D. J. Raine & E. G. Thomas, "Mach's Principle and the Microwave Background," *Astrophysics Letters*, v. 23 (1982), pp. 37–45; D. J. Raine, "Mach's principle and references therein; D. J. Raine, "Mach's principle and references therein; D. J. Raine, Pachart Publishing House, Tuscon 1981.

*priori*, but it is defined by the kinematical, physical invariance group of transformations related to *kinematized* gravito-dynamics.<sup>10</sup>

In this perspective, however, geometry has a foundation completely independent of physics at least at the *non-metrical* level, that is at the *affine* or *projective* geometrical level. It is mathematically constructed in a Platonist world of ideas, on its own specific axioms regarding abstract concepts as points, lines, etc., and only after this stage physics could individuate by a very problematic choice only the kind of metric, that is only the kind of *metrical* geometry to be understood and used only as a physical application of already given mathematical structures.

And even if one understands this determination of metrical geometry by physics in a more radical way as the emergence of a *physical chrono-geometry* as opposed to *mathematical* geometry, it is only the metrical structure of geometry that is physically determined.

Indeed, even if, apart from the Einstein's operational formulation, it was recognized only by Poincaré and Eddington (beyond Whitehead, of course), also special relativity can be interpreted as involving the breakdown of the hierarchy between geometry and physics: here, the problem is the 'elimination' of magnetic forces, and the definition of geometry is given by the kinematical invariance group of transformations related to *partially kinematized* electro-dynamics.<sup>11</sup>

Hence, already special relativity physics replaces a priori geometry with a *chrono-geometry*, but also in this case it is only *metrical* geometry which is determined by physics.

Whitehead, indeed, has solved the greatest question left by Leibniz: relationism actually implies that every concept and every structure within a physical theory must be defined in terms of relations among physical 'elements'; no mathematical or logical concept or structure can be given independently from physical relations. Every other option leads to meta-physics. There is no conventionality of metric.

The fundamental concepts of physics like space and time cannot have any mathematically or logically given *a priori* structure.

<sup>&</sup>lt;sup>10</sup> See, for example: A. O. Barut, *Geometry and Physics. Non-Newtonian Forms of Dynamics*, Bibliopolis, Napoli 1989 and references therein. For the epistemological relevance of this step in the construction of the physical theory, see: D. Finkelstein, "Matter, Space and Logic," *Boston Studies in the Philosophy of Science*, v. 5 (1969), p. 199; E. Giannetto, "On Truth: A Physical Inquiry," in *Atti del Congresso Internazionale "Nuovi Problemi della Logica e della Filosofia della Scienza"*, ed. by C. Cellucci & M. Dalla Chiara, Clueb, Bologna 1991, v. I, pp. 221–228; E. Giannetto, "Note sull'interpretazione della relatività generale di A. S. Eddington," in *Attidell'XICongresso Nazionale di Storia dellaFisica*, ed. by F. Bevilacqua, Goliardica Pavese, Pavia 1993; E. Giannetto, "La logica quantistica trafondamenti della matematica e della fisica", in *Foundations of Mathematics & Physics*, ed. by U. Bartocci & J. P. Wesley, Wesley, Blumberg 1990, pp. 107–127; E. Giannetto, "The Epistemological and Physical Importance of Gödel's Theorems," in *First International Symposium on Gödel's Theorems*, ed. by Z. W. Wolkowski, World Scientific, Singapore 1993, pp. 136-147.

<sup>&</sup>lt;sup>11</sup> E. Giannetto, "Mach's Principle and Whitehead's Relational Formulation of Special Relativity," conference delivered at the *International Congress on 'Mach's Principle,* 'Tübingen, July 1993, in *Proceedings of the Conference on the Physical Interpretations of Relativity Theory III*, London 1994, pp. 126–146.

In Whitehead's formulation of special relativity, physics not only defines the *metrical* geometry, but it also defines *non-metrical, descriptive or projective* geometry, that is geometry *tout court* from its 'foundations.'

From this point of view only, Whitehead's relational chrono-geometry is an actual *physical* geometry, free from any logico-mathematical (Platonist or Kantian, any way idealistic) presuppositions.

Let us consider, first of all, relationism in respect to the fundamental concepts of geometry. Already in a 1906 paper, Whitehead was pointing out that the simplicity of spatial points was in opposition to the relational theory of space: this requires points to be non- fundamental, complex entities.

The statement that the event-particle which one can coordinatizes by four quantities  $(p_1, p_2, p_3, p_4)$  occupies or happens in the point  $(p_1, p_2, p_3)$  means only that the event-particle is only one of the series of event-particles which is the point. That is, point is only a series, a set of physical event-particles.

Hence, a theory of space is not a theory of relations of objects, but of relations of events.

Whitehead explained that in the orthodox theory events are described by means of objects which occupy a dominant position, and so events are considered as a mere play of relations <u>among</u> objects. In this way, space theory becomes a theory of relations among objects instead of relations among events. The consequence is that, for objects are not related to the becoming of events, space as relations among objects is considered as unconnected to time. However, there cannot be space without time, or time without space, or space and time without event becoming.

Thus, at variance with the major part of interpretations of relativity which speak about the spatialization of time, Whitehead obtained a complete *temporalization of space*, so overcoming all the philosophical criticism about that seeming feature of relativity.

Whitehead wrote in *The Principle of Relativity With Applications to Physical Science*:

Position in space is merely the expression of diversity of relations to alternative time-systems. Order in space is merely the reflection into the space of one time-system of the time-orders of alternative time-systems. A plane in space expresses the quality of the locus of intersection of a moment of the time-system in question (call it 'time- system A') with a moment of another time-system (time-system B). The parallelism of planes in the space of time-system A means that these planes result from the intersections of moments A with moments of one other time-system B. A straight line in the space of time-system A perpendicular to the planes due to time-system B is the track in the space of time-system A of a body at rest in the space of time-system B. Thus, the uniform Euclidean geometry of spaces, planeness, parallelism, and perpendicularity are merely expressive of the relations to each other of alternative time-system are also reckoned as parallels. Congruence – and thence, spatial measurements – is defined in terms of the properties of parallelograms

and the symmetry of perpendicularity. Accordingly, position, planes, straight lines, parallelism, perpendicularity, and congruence are expressive of the mutual relations of alternative time-systems...

Let us consider now properly kinematics. Motion is another relation of events, that is a series of events  $(p_1, p_2, p_3, p_4)$  linked to an object, conceived as placed in them, which is defined by its relation with the remaining part of the universe. If one considers another time-system (reference frame), the same motion will appear as a relation of other events  $(q_1, q_2, q_3, q_4)$ , which in general are associated to other different objects.<sup>12</sup>

Hence, even if the motion of one object is relative to the particular considered time-system, such a motion cannot be reduced to an overall rest in any other time-system: that is, it will transform itself into the motion of the *remaining part* of the universe. Indeed, Whitehead kinematized the concept of physical field of an object: it is nothing else than the collection of modifications of event series related to that object: it is a kinematical relation among events and it does not involve any contact or at-a-distance action (his theory of gravitation was not conceived as an action at-a-distance theory as often stated).

### 3. WHITEHEAD'S SPECIAL RELATIVITY

Whitehead's special relativity is constructed by this hierarchical structure: lifeworld experience (experiments too); epistemology and ontology; *Relativistic logic of events; Relativistic set theory of events; Relativistic topology of events; Relativistic non-metrical chrono-geometry; Relativistic metrical chrono-geometry.* 

Thus, what is a material body in general? It is a time series of events, of processes. Nature is the process of all processes interrelated to them. The visible space for us, given the finite speed of light, is not only what happens in our present, but the set of different pasts of all the other processual temporalities of all the other parts of the universe: visible space is the unfolding of different times.

The physics of relativity makes us understand the temporal and processual reality of things and Whitehead's philosophy.

For the relationality of Nature, each part is involved in everything: one part is the set of all relations with the rest of the universe (*togetherness*): it is the relationship with all the other parts, with the otherness that constitutes every part of the universe. *Nature is an inter-related totality*: it is therefore not like a machine, but constitutes a living organism. Every part of Nature is sensitive to the others, every part is alive in different degrees. A *new non-mechanist image of Nature*.

*Process and Reality* can be understood in terms of Whitehead's interpretation of relativity. Whitehead's philosophy of relativity can be used to refute eternalism, four-dimensionalism, perdurantism, endurantism, exdurantism.

<sup>&</sup>lt;sup>12</sup> A. N. Whitehead, *The Principle of Relativity With Applications to Physical Science*, Cambridge University Press, Cambridge1922.

The relational ontology of Nature implies a *cosmic relational ethics*, respectful of all other parts of the universe, of every living part. One new relational image of God as love that grows with always new relationships of the creative process of the universe.

#### BIBLIOGRAPHY

- B. Russell, A Critical Exposition of the Philosophy of Leibniz, G. Allen & Unwin Ltd., London 1900.
- B. Russell, Our Knowledge of the External World, Open Court, La Salle 1914.
- B. Russell, The ABC of Relativity, Harper & Bros., New York and London 1925.
- B. Russell, The Analysis of Matter, G. Allen & Unwin Ltd., London 1927 & Dover, New York 1954.
- A. N. Whitehead, A Treatise on Universal Algebra, with Applications, Cambridge University Press, Cambridge 1898.
- A. N. Whitehead, *The Axioms of Projective Geometry*, Cambridge University Press, Cambridge 1906.
- A. N. Whitehead, *The Axioms of Descriptive Geometry*, Cambridge University Press, Cambridge 1907.
- A. N. Whitehead, *An Introduction to Mathematics*, Williams and Norgate, London & H. Holt and Co., New York 1911.
- A. N. Whitehead & B. Russell (1910–1913), *Principia Mathematica*, Cambridge, Cambridge University Press, v. I, 1910, v. II, 1912, v. III, 1913.
- A. N. Whitehead, *The Organization of Thought Educational and Scientific*, Williams & Norgate, London 1917.
- A. N. Whitehead, Science and the Modern World, The Macmillan Co., New York 1925.
- A. N. Whitehead, Religion in the Making, The Macmillan Co., New York 1926.
- A. N. Whitehead, Symbolism, Its Meaning and Effect, The Macmillan Co., New York 1927.
- A. N. Whitehead, The Aims of Education and Other Essays, The Macmillan Co., New York 1929.
- A. N. Whitehead, The Function of Reason, Princeton University Press, Princeton 1929.
- A. N. Whitehead, Adventures of Ideas, The Macmillan Co., New York 1933.
- A. N. Whitehead, Nature and Life, The University of Chicago Press, Chicago 1934.
- A. N. Whitehead, Modes of Thought, The Macmillan Co., New York 1938.
- A. N. Whitehead, Essays in Science and Philosophy, Philosophical Library, New York 1947.
- A. N. Whitehead, "On Mathematical Concepts of the Material World," *Philosophical Transactions*, Royal Society of London A, v. 205 (1906), pp. 465-525.
- A. N. Whitehead, "La Théorie Relationniste de L'Espace," *Revue de Métaphysique et de Morale*, v. 23 (1916), pp. 423–454.
- A. N. Whitehead, "Space, Time, and Relativity," in *Proceedings of the Aristotelian Society*, v. 16 (1915–1916), pp. 104-129.
- A. N. Whitehead, An Enquiry on the Principles of Natural Knowledge, Cambridge University Press, Cambridge 1919.
- A. N. Whitehead, The Concept of Nature, Cambridge University Press, Cambridge 1920.
- A. N. Whitehead, *The Principle of Relativity With Applications to Physical Science*, Cambridge University Press, Cambridge 1922.
- A. N. Whitehead, *Process and Reality. An Essay in Cosmology*, The Macmillan Co., New York, 1929.
- *The Harvard Lectures of Alfred North Whitehead*, 1924–1925: Philosophical Presuppositions of Science, ed. by Paul A. Bogaard, Jason Bell, Edinburgh University Press, Edinburgh 2017.
- The Harvard Lectures of Alfred North Whitehead, 1925–1927: General Metaphysical Problems of Science, ed. by Brian G. Henning; Joseph Petek; George Lucas, Edinburgh University Press, Edinburgh 2021.