MATHEMATICAL INFINITY AND THE ENHANCED INDISPENSABILITY ARGUMENT

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Abstract. This paper examines the ontological status of mathematical infinity in the context of the Enhanced Indispensability Argument. While the classical Quine–Putnam Indispensability Argument grounds realism about mathematical entities in their indispensability to empirical science, the enhanced version emphasizes their genuine explanatory role. I argue that if explanatory indispensability is sufficient to justify commitment to mathematical entities, then it equally justifies commitment to mathematical infinity. Far from being a mere idealization or heuristic device, infinity plays an indispensable role in explaining certain empirical phenomena. Consequently, the Enhanced Indispensability Argument supports a realist interpretation of mathematical infinity and challenges views that regard it as a purely instrumental or fictional notion. This discussion situates the argument within broader debates on the ontological and epistemological status of abstract entities in scientific explanation.

Keywords: Mathematical Infinity, Enhanced Indispensability Argument, Ontological Commitment, Scientific Explanation, Mathematical Platonism.

1. INTRODUCTION

The relationship between mathematics and reality is one of the significant issues addressed by contemporary authors within the philosophy of mathematics.[1] Although, in everyday scientific practice, mathematics is an indispensable component of physical theories, philosophical analysis remains confronted with the question: what exactly commits us to believing in the existence of mathematical entities?

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[1] See, for example, Mark Balaguer, *Mathematical Anti-Realism and Modal Nothingism*, Cambridge, Cambridge University Press, 2023; Philip Bold, "Later Wittgenstein on 'Truth' and Realism in Mathematics", *Philosophy*, vol. 99, nr. 1, 2024, pp. 27–51; Alan Weir, "Putnam, Gödel, and Mathematical Realism Revisited", *International Journal of Philosophical Studies*. vol 32, 2024, pp. 146–168.

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The classical *Indispensability Argument* (IA), formulated by Quine and Putnam, maintains that we ought to believe in the existence of mathematical objects because they are indispensable to the formulation of our best empirical theories.[2] However, a more recent formulation of the argument, known as the *Enhanced Indispensability Argument* (EIA), goes a step further – toward the idea that mathematics not only enables the description of nature but also plays *an explanatory role* in scientific explanations.[3]

Within this new framework, the concept of mathematical infinity presents a particular challenge. Infinite entities – such as infinite sets, real numbers, or Cantorian cardinals – lack direct empirical correlates, yet they are integral to a significant number of mathematical theories used in science.[4] This raises the question: if infinity is unavoidable in our mathematical models, are we, according to the EIA, committed to believing in the existence of infinite objects? Or, on the contrary, is infinity an example of the limit beyond which this argument can be sustained?

This paper defends the thesis that the Enhanced Indispensability Argument, if accepted as a valid Platonist argument, can justify an ontological commitment to mathematical infinity, since the concept of infinity possesses an explanatory function with respect to certain empirical situations. Although infinity is conceptually indispensable within mathematical formalism, its role is primarily epistemic – it enables the theoretical framing of natural phenomena by providing direct formal support for both applicable and non-applicable mathematical theories. We will attempt to show that mathematical infinity directly contributes to the explanation of empirical phenomena in the same sense in which certain other mathematical entities and theories do. This would demonstrate that the EIA, which justifies Platonism in mathematics insofar as mathematical structures have explanatory power, does not leave infinity as a philosophically "idealized" concept, but rather grants it ontological status by assigning it to the small set of mathematical entities for which such a status is secured.

The paper is structured into five parts. Following the introduction, the second section presents the theoretical framework for two types of indispensability, IA and EIA. The third section analyzes two kinds of mathematical infinity – potential and actual. The fourth part addresses the status of mathematical infinity in the context of EIA, followed by a concluding section.

[2] Alexander C. Paseau and Alan Baker, *Indispensability*, Cambridge, Cambridge University Press, 2023.

[3] Robert Knowles, "Platonic Relations and Mathematical Explanations", *Philosophical Quarterly*, vol. 71, 2021, pp. 623–644.

[4] Mathematical analysis, number theory, etc.

2. THEORETICAL FRAMEWORK: INDISPENSABILITY AND ENHANCED INDISPENSABILITY

The question of the ontological status of mathematical entities occupies a central place in the philosophy of mathematics, particularly in contemporary debates on realism and anti-realism.[5] One of the most influential arguments in favor of mathematical realism is the so-called *Indispensability Argument* (IA), whose classical formulation is attributed to Quine and Putnam. In its simplest form, this argument proceeds from scientific realism – the view according to which we are ontologically committed to the existence of the entities posited by our best scientific theories, precisely because we regard those theories as true, or at least approximately true. If mathematical structures are indispensable to the formulation and application of such theories, then, by analogy, they too ought to be granted ontological status. One possible formulation of the so-called Quine–Putnam argument may take the following form:

(1) We ought to have ontological commitment to all and only those entities that are indispensable to our best scientific theories.

(2) Mathematical entities are indispensable to our best scientific theories.

Therefore:

(3) We ought to have ontological commitment to mathematical entities.[6]

By adopting this approach, Quine and Putnam reject the possibility of treating mathematical entities merely as fictional or purely formal instruments. According to their view, if mathematical objects cannot be eliminated from our most successful scientific explanations, then we must relate to them with the same degree of ontological commitment as we do to electrons, genes, or black holes. Quine's well-known maxim – "To be is, purely and simply, to be the value of a variable"[7] – is intended to be understood in a context far broader than a narrowly logical one, namely that entities quantified over within our best-confirmed scientific theories must be included in our ontological picture of the world.[8] In this way, mathematics, as an integral component of scientific theories, acquires ontological "weight."

Nevertheless, the classical Indispensability Argument has faced a range of criticisms since the 1980s. The most significant of these – both historically and in their contemporary formulations – have emerged from fictionalist and nominalist

[5] See, for instance, Guanglong Luo, "Nominalism and Mathematical Objectivity", *Axiomathes*, vol. 32, nr. 3, 2022, pp. 833-851; Mahdi Khalili, "Entity Realism Meets Perspectivism", *Acta Analytica*, vol. 39, 2024, pp. 79–95; Silvia Jonas, "Mathematical Pluralism and Indispensability", *Erkenntnis* vol. 89, 2023, pp. 2899–2923.

[6] Mark Colyvan, *The Indispensability of Mathematics*, New York, Oxford University Press, 2001, here p. 11.

[7] Willard V. Quine, "On What There Is", *The Review of Metaphysics*, vol. 2, nr. 5, 1948, pp. 21–38, here p. 32.

[8] Ibid., pp. 32–33.

positions.[9] Field, as a pioneer of this line of thought, argued that scientific theories can, at least in principle, be reformulated without reference to abstract mathematical objects, while preserving their empirical content intact (Field, 2016, p. 3).[10] If such an elimination were possible, the claim that we are ontologically committed to mathematics would lose its foundation. It can therefore be said that the attempt to respond to these criticisms constitutes one of the main motivations behind the emergence of a modernized version of the argument – the Enhanced Indispensability Argument (EIA). What distinguishes the EIA from the classical IA is its emphasis on the idea that mathematics does not function in scientific theories merely as a formal tool for calculation or representation but rather plays an autonomous explanatory role. In other words, the EIA focuses on cases in which the explanation of a physical phenomenon would be incomplete, or even impossible, without appeal to mathematical structures. However, the literature contains relatively few examples in which mathematical tools are genuinely employed to explain physical phenomena. Among the most frequently cited examples are the Königsberg bridges problem, the Honeycomb Theorem, and the case of the North American periodical cicadas (*Magicicada*).[11]

The formulation of the EIA most commonly encountered in contemporary literature was provided by Alan Baker in the form of a kind of modal syllogism:

(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.
(2) Mathematical objects play an indispensable explanatory role in science.
(3) Hence, we ought rationally to believe in the existence of mathematical objects.[12]

Within the framework of the EIA, the relationship between mathematics and science thus becomes bidirectional: mathematics not only enables the formulation of natural laws, but also contributes to our understanding of *why* certain phenomena occur and *how* specific empirical problems are resolved. From this follows a stronger ontological claim: if mathematics can explain empirical facts, then its entities must possess some form of real existence – at least in the sense of abstract yet genuinely existing structures. However, this idea also imposes significant constraints. If the justification for ontological commitment to mathematics is

[9] Hartry H. Field, *Science Without Numbers: A Defense of Nominalism*, New York, Oxford University Press, 2016; Jody Azzouni, *Deflating existential consequence: A case for nominalism.* New York, Oxford University Press, 2004; Otavio Bueno, "*Mathematical Fictionalism Revisited"*, in: Cristian Soto (eds.), *Current Debates in Philosophy of Science: In Honor of Roberto Torretti*, Cham, Springer Cham, 2023, pp. 103–122; Jessica Carter, "Mathematical Practice, Fictionalism and Social Ontology", *Topoi*, vol. 42, 2022, pp. 211–220.

[10] H. Field, *Science Without Numbers: A Defense of Nominalism*, P-3.

[11] Mark Colyvan, "The Ins and Outs of Mathematical Explanation", *The Mathematical Intelligencer*, vol. 40, nr. 4, 2018, pp. 26–29.

[12] Alan Baker, "Mathematical Explanation in Science", *British Journal of Philosophy of Science*, vol. 60, nr. 3, 2009, pp. 611–633, here p. 613.

grounded in its explanatory role, then each individual mathematical construction must be assessed according to *whether it genuinely contributes to the explanation of the empirical world*. It is precisely at this point that the notion of mathematical infinity becomes a crucial test case for the EIA. Infinite entities occupy a central position in modern mathematics, yet it is far from clear whether they can be shown to have any explanatory function within the empirical sciences. This raises the question of whether the EIA can warrant a realist commitment to infinite structures, or whether infinity instead marks a boundary beyond which the argument loses its force.

3. ON TWO CONCEPTIONS OF (MATHEMATICAL) INFINITY

The concept of infinity has, throughout history, attracted sustained and significant attention from various kinds of thinkers. The reasons for this interest can be traced to metaphysical and philosophical considerations, religious motivations, as well as strictly exact and mathematical concerns. When addressing the notion of infinity within the framework of the philosophy of mathematics, Aristotle's *Physics* occupies an indispensable position, as it provides one of the earliest systematic analyses of infinity and establishes conceptual distinctions that have profoundly influenced subsequent philosophical and mathematical discussions:

> The infinite exhibits itself in different ways – in time, in the generations of man, and in the division of magnitudes. For generally the infinite has this mode of existence: one thing is always being taken after another, and each thing that is taken is always finite, but always different. Again, 'being' has more than one sense, so that we must not regard the infinite as a 'this', such as a man or a horse, but must suppose it to exist in the sense in which we speak of the day or the games as existing things whose being has not come to them like that of a substance, but consists in a process of coming to be or passing away; definite if you like at each stage, yet always different.[13]

Aristotle's understanding of the concept of infinity constituted the foundation upon which two distinct approaches to this notion developed throughout history. One is the so-called potential infinity, which Aristotle regarded as the only admissible form, while the other is the so-called actual infinity, which came to be predominantly employed within the scientific community following Cantor's establishment of set theory at the end of the nineteenth century. Although Cantor's work, and subsequently the axiomatic ZF set theory, consolidated the contemporary dominance of the latter conception and largely relegated the ancient understanding of infinity to domains more closely associated with religious and philosophical

[13] Aristotle, *Physics*, translated by R.Waterfield (D. Bostock, eds.), Oxford, Oxford University Press, 1999, here pp. 210–11.

interpretations, it can nonetheless be argued that a certain form of "tension" between these two conceptions persists even today.

In the spirit of the idea articulated in the *Physics*, one may say that a concrete manifestation of infinity is given, for example, by the infinite sequence ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ...), that is, it is represented by the set $A = \{ \frac{1}{2n}, n \in \{1, 2, 3, \ldots\}\}$, or equivalently by the set $B = \{\frac{1}{2n}, n \in \mathrm{N}\}$. The elements of the sets *A* and *B* are precisely the members of the aforementioned sequence. However, what do we actually know about this sequence, and are its elements explicitly listed? Strictly speaking, not all members of the sequence are listed; rather, only a finite number of its initial terms are provided, after which – following standard mathematical practice – it is assumed that there exists a well-defined procedure by means of which any term of the sequence can be determined. For mathematicians, there is no particular concern regarding the (im)possibility of writing down or enumerating all elements of an infinite sequence. More precisely, their concern in this respect is no greater than it is with regard to the possibility of enumerating the elements of a finite sequence, such as (1, 2, 3, …,100 000). In a similar manner, mathematicians, drawing on the achievements of Cantorian set theory, shape their approach to both finite and infinite sets. For instance, one may order the elements of the set *B* just as one may order the elements of the set $C = \{1, 2, 3, \ldots, 100{,}000\}$, speak of the number of elements in each set, investigate their possible relations, and characterize them in various other ways.

The majority of mathematicians approach the concept of infinity, that is, the notion of an infinite set, in a routine manner, without engaging in any explicit metaphysical, ontological, or epistemological problematization. The attitude toward infinity, for example as expressed in the assertion

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e \tag{1}$$

is of an exclusively operational and technical nature. Equality (1) asserts that a certain expression attains a specific value as the variable *n* in that expression tends toward infinity. *The concept* of infinity is thus "encapsulated" in the symbol ∞, while the progression of the variable *n* toward infinity is denoted by the notation $n \to \infty$. The value of the expression $\left(1+\frac{1}{n}\right)^{n}$ will be closer to the value *e* the larger the value of the variable *n* is, or, more precisely, the closer it is to infinity. In this respect, the intuition associated with the meaning of the symbol (or concept) ∞ is considerably more obscure than the intuition we have regarding the meaning of most other mathematical symbols (or concepts), such as the symbol of the natural number 3 or the symbol of a function *f(x)*.

What may appear problematic in the understanding of equality (1) is the fact that, within it, the variable *n* is said to tend toward infinity. The variable *n* never reaches infinity, regardless of how much it is increased, which is a trivial consequence of the construction of the set of natural numbers.[14] If we wish to be formally precise, we must say that the expression on the left-hand side of equality (1) will not attain the value *e* no matter how "large" the value assigned to the variable *n* may be. As *n*, a natural number, increases, the value of the expression on the left-hand side of equality (1) approaches the value *e*, yet it never becomes equal to *e*, regardless of how far one proceeds in the successive steps of increasing *n*. In other words, the limit in equality (1) could be interpreted as follows: if the variable *n* were to reach infinity – whatever that notion may mean – then the expression on the left-hand side of the equality would take on the value *e*. However, how is one to speak meaningfully about what would happen to the expression in question if the variable *n* were to reach infinity, when no variable has ever reached such a value in the calculations of mathematicians throughout the entire history of mathematics, nor can it be said that any mathematician has provided an exact description, by means of a formal tool, of such a value in the way this has been done, for example, for finite cardinals?

Mathematical reasoning in the preceding case exhibits a character that may be described as a form of *certain*, or *reliable*, scientific induction. Indeed, one could say that mathematicians discern a regularity manifested in formal certainty, which justifies the assertion that, for each successive natural number *n*, the value of the expression $\left(1+\frac{1}{n}\right)^n$ is closer to the value *e* than it was for the preceding value of *n*. More formally, for an arbitrarily chosen $\varepsilon > 0$, however close one wishes to be to the value *e*, it is possible to find a natural number n_0 such that for every $n \geq n_0$, the following holds:

$$\left|\left(1+\frac{1}{n}\right)^n - e\right| < \varepsilon \, .$$

In other words, all members of the sequence $\left(1+\frac{1}{n}\right)^n$, $n \in N$, with the possible exception of finitely many, lie within an arbitrarily small neighborhood of the number *e* on the real line *R*. This is the reason why one speaks of *a certain*, or *indisputable*, form of scientific induction. The regularity observed in a finite number of cases unquestionably applies to all other mathematical objects, including infinitely many of them, and there exists no moment of uncertainty in the "behavior" of the remaining mathematical entities to which the assertion pertains. This understanding is not altered by the historical fact that the mathematical

[14] See, for instance, Edward Scheinerman, *From Counting to Continuum: Natural Numbers, Integers, and the Rich Structure of Number Systems*, Cambridge, Cambridge University Press, 2024.

community has, throughout the development of mathematics, operated with only finitely many such entities – in this case, natural numbers. No matter how close one wishes to approach the number *e*, this can be accomplished in practice by choosing a sufficiently large natural number *n*. On this basis, the mathematician concludes that the limit in equality (1) is the number *e*. But is such reasoning justified? Is the mere fact that an object can be approached arbitrarily closely, depending on the increase of a variable *n*, sufficient grounds to assert that this object can actually be attained?

At this point, it is necessary to return to the distinction between potential and actual infinity, that is, to the differentiation of the conception of infinity, on the one hand, as *a process*, and, on the other hand, as *a set*. The tradition following Aristotle understood mathematical infinity as a process without end, in which new mathematical entities – previously unobserved – can continuously be discerned. By contrast, Cantor’s theory gave rise to refined achievements in which infinity is treated as a distinct mathematical *object*, namely as a set of infinitely many individual entities. Such a theory made it possible to distinguish different types of infinite sets, and, consequently, different types of transfinite ordinals and cardinals corresponding to these sets. Examples of potential infinity in mathematics are ubiquitous. We encounter it when enumerating the natural numbers, when imagining regular polygons with ever-increasing numbers of sides, or when attempting to write out the decimals of $\sqrt{5}$, among others. In such cases, infinity is understood as a kind of movement, an unfinished process, without any epistemological obstacles. We can witness this movement by following the process for as long as we wish, analyzing it in detail and identifying each iteration we choose. Infinity in this sense is a continuous flow that takes the form of an incomplete state. Actual infinity, by contrast, transforms this understanding and represents infinity conceptually as *a thing*, *an object*, *an almost completed state*. This understanding is exemplified when we speak of the set *Q* of rational numbers or the set *R* of points on the real line. The concept of actual infinity does not exclude the notion of potential infinity, but it emphasizes the reification, the conceptual “encapsulation,” of potential infinity. In this way, it becomes operationally easier to work with such forms of infinity: all entities representing this type of infinity can be compared with one another, their hierarchy can be discussed, and they can be related to corresponding transfinite numbers, and so forth.

The methodology of epistemologically “taming” infinity – of creating an embodied, objectified form of an infinite process – gives rise to certain cognitive, and thereby ontological, uncertainties regarding the notion of actual infinity. Indeed, it is not a trivial matter to understand how a concept that implies a continuous, unbroken process can be apprehended and comprehended as a thing. One attempt to explain this issue is associated with the so-called *metaphorical* understanding of actual infinity:

> … If a process has no end, there can be no 'ultimate result.' But the mechanism of metaphor allows us to conceptualize the "result" of an infinite process – in the only way we have for conceptualizing the result of a process – that is, in terms of a process that does have an end.
> We hypothesize that all cases of actual infinity – infinite sets, points at infinity, limits of infinite series, infinite intersections, least upper bounds – are special cases of a single general conceptual metaphor in which processes that go on indefinitely are conceptualized as having an end and an ultimate result…[15]

In other words, the understanding of actual infinity should be construed as built upon a metaphor – a kind of abbreviated comparison with a situation that exists within the context of potential infinity. However, it remains unclear how this metaphor actually functions. How can an infinite process be replaced and represented by a completed and "graspable" object that, although fully shaped and devoid of signs of infinity, serves as a kind of personification of the infinite process? One possible answer is that the objectification of an infinite process can be metaphorically represented through *the principle* that governs the process. This principle is, in essence, a pattern or formula that explicitly determines an arbitrary member of the infinite process, thereby allowing any member of the process to be explicitly identified. By expressing such a principle, we represent all elements of the infinite process, even though cognitive agents will directly perceive only finitely many of them. For example, Euler's function φ maps an arbitrary natural number m to the number of integers from 0 to $m - 1$ that are relatively prime to m. For instance, $\varphi(1) = \varphi(2) = 1$, $\varphi(3) = \varphi(4) = 2$, $\varphi(5) = 4$.[16] In this way, we have precisely defined an infinite process, the sequence of natural numbers

$$1, 1, 2, 2, 4, \ldots$$

in which there exists a clear principle by which any term of the sequence can be unambiguously determined through a finite procedure. In this case, the principle underlying the sequence functions as a kind of metaphor – a paradigm that personifies and represents the entire sequence.

A second possible answer is finitistically oriented. Since a metaphor represents a kind of abbreviated comparison – a non-literal alignment of structurally distinct concepts – it may be said that finitistic attempts at explanation in this context are natural. Accordingly, some authors return the idea of the metaphor to finitistic foundations,[17] seeking to interpret the practical circumstances related to the early

[15] George Lakoff and Rafael E. Núñez, *Where mathematics comes from*, New York, Basic Books, 2000, here p. 158.

[16] See, for example, Paul Erdos and Janos Suranyi, *Topics in the Theory of Numbers*, USA, Springer, 2003, here p. 58.

[17] Markus Pantsar, "In Search of $\aleph_0$: How Infinity Can Be Created", *Synthese*, vol. 192, nr. 8, 2015, pp. 2489–2511, here p. 2509.

acquisition of children's mathematical concepts. In this way, they attempt to explain how, through a process involving specific concrete entities, a new entity of a different quality arises metaphorically, emerging from abstraction from the initial process. For example, children reach number eight gradually, most often by counting fingers one by one, until all eight fingers have been counted. *But the end product of the counting is not the eight extended fingers. It is the quantity of eight.* In this case, the finite outcome is the result of a determined number of iterations, yet the final product continues to exist independently of the possibility of observing such iterations again.

4. INFINITY AS A TEST FOR THE ENHANCED INDISPENSABILITY ARGUMENT

The concept of actual mathematical infinity raises new philosophical challenges related to the epistemic–ontological obstacles that arise in connection with its comprehension. The overall context is rendered more complex not only by metaphorical approaches to solving the problem that are finitistic in character – such as the one mentioned at the end of the previous section – but also by ontological analyses that identify Platonism with realism.[18] Moreover, the difficulty of understanding the concept of infinity is further compounded by more recent materialist analyses that imperatively condition the ontological status of entities on a finitistic approach.[19] Could traditional or contemporary mathematical Platonism consistently provide a framework for the notion of actual infinity, and if so, in what way? How might we accept the central Platonist claim concerning the existence of mathematical objects when infinity is at issue? As is well known, according to *the traditional Platonist* standpoint, all objects encountered in the physical world have their originals in a distinct world of ideas.[20] In this Platonic world reside the primordial sources, archetypes, and forms of all entities, including the mathematical entities we encounter in our world. The notion of form also appears in discussions of Cantor's understanding of actual infinity. Cantor held that transfinite numbers are *forms* or modifications of the actual infinite.[21] However, this notion of form does not have the same meaning as Plato's concept. Cantor's forms of infinity are

[18] Penelope Maddy, *Realism in Mathematics*, New York, Oxford University Press, 1990.

[19] Ozan E. Derin and Bekir Baytaş, "The Architecture of Relational Materialism: A Categorial Formation of Onto-Epistemological Premises", *Foundations of Science*, 2025, https://doi.org/10.1007/s10699-025-09977-0

[20] Anna Marmodoro, *Forms and structure in Plato's metaphysics*, Oxford, Oxford University Press, 2021.

[21] Georg Cantor, *LogischPhilosophische Abhandlung*, in E. Zermelo (Eds.), *Gesammelte Abhandlungen mathematischen und philosophischen inhalts*, Hildesheim, Georg Olms Verlagsbuchhandlung, 1932, here pp. 395–396.

rather manifestations or modes of appearance of infinity in our world; that is, they are not originals (Platonic forms) situated in a separate metaphysical realm. Cantor's forms, such as $\aleph_0$ or $\aleph_1$, are studied by mathematicians within this world, just as they study actual infinity as embodied, for example, in the set $\mathbb{N}$. Thus, in Cantor's sense, instances of actual infinity, as well as its forms, are objects of our world rather than of some distant metaphysical realm of the sort posited in Plato's construction. On the other hand, Cantor's reflections on infinity also contain certain religious and metaphysical elements. It may be said that he believed his analyses of infinity, in a certain way, to lead toward God, a belief that directly influenced his theological reflections toward the end of his life.[22] This constitutes a metaphysical point at which similarities between Platonic and Cantorian treatments of infinity may be sought, while at the same time marking a point of divergence in their respective metaphysical approaches. On the one hand, Cantor's conception does not presuppose a distinct metaphysical realm of archetypes of the kind found in Plato; on the other hand, Plato's conception does not involve a predominantly monotheistic view of God, which is closest to Cantor's standpoint.[23]

Let us now consider the extent to which *contemporary* forms of justification for mathematical Platonism are consistent with the notion of actual infinity. Contemporary justifications of mathematical Platonism depart from traditional ones primarily by abandoning arguments that had an explicitly metaphysical character. Accordingly, within the philosophical frameworks of the twentieth and twenty-first centuries, the focus is placed on the IA and EIA arguments, which were recalled in Section 2. The EIA maintains that mathematical entities deserve ontological commitment insofar as they play an indispensable explanatory role in scientific theories. While many mathematical structures are evidently useful in this capacity – for instance, matrices in quantum mechanics or differential equations in fluid dynamics – the notion of infinity constitutes a critical test for both the strength and the scope of this argument. At present, there exist several analyses in the literature that either challenge or support the EIA.[24] Without, at this stage, questioning the validity of the argument itself, our aim is to examine to what extent, assuming its validity, it could be treated as a reliable tool for securing the ontological status of mathematical infinity. So far, only a small number of mathematical entities can be found in the literature for which there exist analyses allowing us to claim that they are explanatorily indispensable to the account of

[22] M. Pantsar, "In Search of $\aleph_0$: How Infinity Can Be Created", p. 2490.

[23] Rico Gutschmidt and Merlin Carl, "The negative theology of absolute infinity: Cantor, mathematics, and humility", *International Journal for Philosophy of Religion*, vol. 95, nr. 3, 2024, pp. 233–256.

[24] Daniele Molinini, Fabrice Pataut, & Andrea Sereni (eds.), *Indispensability and explanation* [Special issue]. *Synthese*, vol. 193, nr. 2, 2016; Vladimir Drekalović, "Is the Enhanced Indispensability Argument a Useful Tool in the Hands of Platonists?", *Philosophia*, vol. 47, 2019, pp. 1111–1126.

physical phenomena and that, on the basis of the EIA, their ontological status is thereby secured.[25] Our objective is to investigate whether such a limited class could gradually be expanded to include, among other things, infinite objects. In other words, we wish to explore the possibility that mathematical infinity might be treated as an explanatorily indispensable entity in the explanation of certain physical phenomena. To this end, it would be necessary to identify at least one case in which actual infinity plays the role of an explanatorily indispensable mathematical entity in connection with the explanation of a physical phenomenon or the resolution of an empirical problem.

Considering the literature in physics currently available in which the application of infinite mathematical entities occurs, we cannot claim that the ontological status of infinity is secured by appealing to the EIA. Namely, infinite objects (such as sequences, series, and the like) in physics are, generally speaking and on the basis of the published material to date, employed as *epistemic tools*: they facilitate calculation and the structuring of theory, but they do not play a genuinely explanatory role. For example, infinite series are used in *the description* of field theory and gravitation by being employed in the solution of differential equations in curved space–times, where the functions appearing in the equations are expanded into infinite series and used to obtain solutions that otherwise lack a closed form.[26] Infinite series are also used, for instance, to obtain analytic solutions of differential equations in physical models (e.g., heat, waves, nonlinear systems).[27] However, even in these cases, such infinite entities do not assume a role that could properly be described as explanatory.

The distinction between the descriptive (non-explanatory) and the explanatory roles of mathematical objects that appear in certain accounts of physical phenomena and in the resolution of various empirical problems is not easily drawn. The kinds of roles in question are highly heterogeneous and resist any clear-cut or discrete classification into explanatory and non-explanatory ones. Indeed, there are cases in which this distinction is obvious and their categorization straightforward. For example, when we use real numbers to express the magnitude of a given physical quantity, such as the temperature at which a liquid boils, it is trivially clear that the mathematical objects involved – in this case, real numbers – do not explain the

[25] These include, for instance, objects from number theory (as exemplified by the case of North American cicadas, *Magicicada*), objects from graph theory (as exemplified by Euler's problem inspired by the case of the Seven Bridges of Königsberg), and objects from theories of optimality (as exemplified by the Honeycomb Problem), among others. For further discussion, see, for example, M. Colyvan, "The Ins and Outs of Mathematical Explanation", *The Mathematical Intelligencer* and Vladimir Drekalović, "Mathematical Explanation as Part of an (Im)perfect Scientific Explanation: An Analysis of Two Examples", *Filozofia Nauki*, vol. 27, nr. 4, 2019, pp. 23–41.

[26] Emir Baysazan, Tolga Birkandan and İsmail E. Ünver, "Analysis of Scalar Fields with Series Convolution", *European Physical Journal C*, vol. 84, nr. 10, 2024, pp. 1–19.

[27] Angela Slavova (ed.), *New Trends in the Applications of Differential Equations in Sciences*, Cham, Springer, 2024.

phenomenon of boiling. They merely serve as instruments for describing a certain thermal state of the liquid. By contrast, when, within the framework of graph theory, we demonstrate the impossibility of a certain kind of traversal through a given graph, after having mapped the concrete situation in Königsberg onto that graph, it is evident that mathematical resources are being used to explain salient features of a specific empirical problem.

Nevertheless, in the majority of cases the distinction between the explanatory and the non-explanatory roles of mathematical objects in explanations of physical phenomena and in the resolution of empirical problems is far from trivial. To illustrate this point, let us consider the well-known problem of calculating the measure of the area under a curve in the plane.[28] This can be regarded as a concrete case of solving an empirical task, namely, the precise determination of the area bounded by an arbitrary curve. As is well known from the history of mathematics, this problem was effectively resolved through the introduction of the idea of the definite integral, which emerged from the independent results obtained by Newton and Leibniz in the second half of the seventeenth century.[29] Leibniz's approach to this problem was grounded in the more general idea that the area bounded in the plane by a curve can be approximately equated with the area of a corresponding polygon, and that it can be exactly equated with the area of a corresponding polygon with *infinitely* many sides.[30] Reasoning in a similar manner within the coordinate plane, Leibniz conceived of the area between an arbitrary curve and the *x*-axis on the interval [a, b] as the sum of *infinitely* many *infinitesimal* rectangles. This construction lies at the foundation of the modern conceptions of the Riemann and Lebesgue integrals.[31] Unlike the previously mentioned examples, in this case the discussion of the role played by mathematical entities in the explanation or solution of a concrete empirical problem is not straightforward. Indeed, what is the role of infinity in the resolution of this particular empirical problem? Does the use of an infinite sum or of infinitesimal rectangles *explain* the possibility of calculating the measure of a specific area in the empirical world?

In the problem of the Königsberg bridges, the question concerning the empirical world was whether it is possible, by adopting an appropriate strategy, to traverse all the bridges of Königsberg in such a way that each bridge is crossed

[28] Although this problem can be considered within a purely mathematical setting, it is clear that it can also be understood as an empirical problem, for instance, as the calculation of the measure of a concrete surface in the physical world.

[29] Certain results relevant to this topic were already obtained by Archimedes in the third century BCE (Thomas L. Heath (trans./ed.), *The Works of Archimedes*, Cambridge, Cambridge University Press, 2002), but our discussion will be confined to the framework of modern mathematics.

[30] Eberhard Knobloch, "Analiticidad, Equipolencia y Teoría de Curvas en Leibniz", *Llull: Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas*, vol. 36, 2013, pp. 283–306, here p. 303.

[31] Yoshifumi Mimura, *The Mimura Integral: A Unified Framework for Riemann and Lebesgue Integration*, 2025, Preprint retrieved from arXiv: 2509.25875.

exactly once. In this case, a negative answer was provided on the basis of mathematical considerations concerning the structure of graph theory, into which the concrete empirical structure was mapped in a relevant manner. These mathematical considerations are encapsulated in what is known as Euler's theorem. In contemporary literature, this case is commonly cited as a paradigmatic example in which mathematical objects play *an explanatory role* with respect to a problem drawn from physical reality. Similarly, the problem of calculating a definite integral may be regarded as content that is transferred in a relevant way from empirical reality, namely, the determination of the area bounded by a curve that is not rectilinear. In the latter case, the modifications of the entities involved in the transition from the original empirical structure to a purely mathematical one are even less substantial than in the case of the Königsberg bridges. Since the role of graph theory is unambiguously explanatory in the case of Euler's problem, the same must be said of the entire mathematical apparatus, including infinite entities, that is employed in solving the practical problem addressed by the idea of the Leibnizian integral. Hence, infinite objects have an explanatory role in this concrete empirical problem.

In order to speak, on the basis of the EIA, about the ontological status of mathematical objects, it is necessary that they possess, in addition to an explanatory role, an indispensable role in science. Do infinite objects play such an indispensable role in concrete cases? In other words, is the problem of determining the area under a curve exactly solvable without recourse to infinite entities, sequences, or series? At the current stage of mathematical development, the answer would be conditionally affirmative. That is, the computation of such an area is possible, but not always and not for all curves. For certain trivial curves or functions, the area beneath the curve can be calculated purely geometrically, or by some other method, without any use of infinite series or sequences. For example, when the function is a straight line or a circular arc, the area can be determined using standard formulas for the area of a triangle, trapezoid, or circle. Similarly, when the curve can be expressed as a function f possessing an elementary antiderivative F, the area may be calculated using the definite integral, without the explicit use of infinite sequences or series:

$$P = \int_a^b f(x)dx = F(b) - F(a)$$

Examples of such functions include polynomials, exponential functions, trigonometric functions, and certain rational functions. In these cases, infinite entities do not explicitly appear in the calculation itself, although the theoretical foundation of the integral is tied to limiting processes. However, for a large number of functions, the computation of area is not possible without infinite processes or objects (depending on whether infinity is conceived in a potential or actual sense).

For instance, this is not feasible for the functions e^{-x^2} or $\frac{\sin x}{x}$. Thus, in many practical cases, the area can be computed without recourse to infinite series or sequences. Yet, in the general case, the problem of determining the area under a curve unavoidably involves considerations of limits and the use of infinite processes. Employment of infinity is indispensable because, intuitively, we are attempting to solve the problem of precisely measuring something that cannot be decomposed into a finite number of perfectly known parts. Accordingly, infinite mathematical objects satisfy condition (2) of the EIA, since, at the current stage of development of mathematical theories, they perform an indispensable explanatory role in empirical science.

5. CONCLUSION

The aim of this paper was to investigate the relationship between the notion of mathematical infinity and the EIA. Through an analysis of both the classical and the enhanced arguments of indispensability, as well as historical and contemporary treatments of infinity, key tensions between the epistemic functionality and the ontological status of infinite entities have been highlighted. The classical IA justifies realism regarding mathematical entities on the basis of their indispensability to our best empirical theories. EIA, by contrast, extends this justification with the additional requirement that mathematics in scientific theories plays an explanatory role – that it does not merely serve as an instrument, but helps to explain why certain phenomena occur.

The notion of infinity presents a test case for EIA. Infinite entities, such as infinite sets or continua, while fundamental to formal mathematics, rarely have a direct explanatory role in the empirical sciences. They facilitate modeling and the formalization of theories but generally do not contribute to the explanation of phenomena in the same way as other mathematical objects. Nevertheless, using the example of integral calculus, we have attempted to show that, at the current stage of development of mathematical theories, some infinite structures satisfy the conditions imposed by EIA's second premise. On this basis, we can assert that EIA secures an ontological status for such structures. This conclusion, however, is conditional to the extent that the strength of EIA itself is contingent and limited, given the general weaknesses shared by both IA and EIA regarding the problem of future events. Specifically, "being indispensable," unlike "being explanatory," is a property that can be attributed to mathematical objects only conditionally, in the context of current mathematical achievements and the edifice of mathematical theories as it exists at present. No one can predict the scope in which this edifice will be further developed, and thus cannot anticipate the objects that will comprise it. This further implies that the characteristic of "indispensability" with respect to the explanation of a physical phenomenon or the solution of an empirical problem

P cannot be guaranteed for any mathematical object *O*, since we cannot assert with certainty that in the future an object O_1 will not be defined, which could also serve to explain the phenomenon or solve the problem *P*. This burden, inherent in EIA, must also be borne by the ontological status of infinite mathematical objects.